Moving Crocus into the Richards equation, away from the bucket model !?

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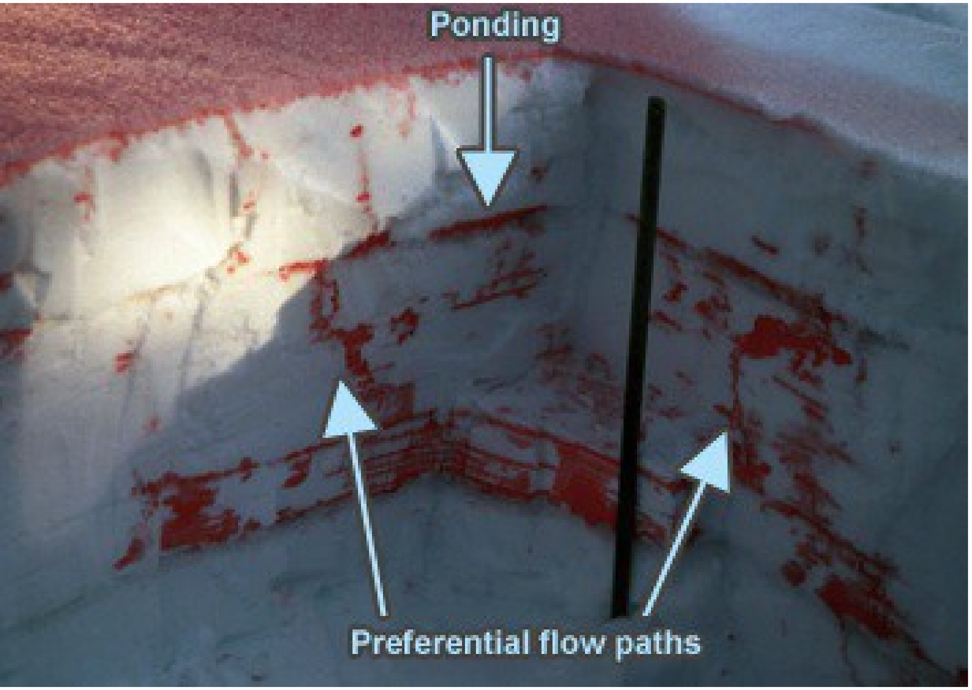
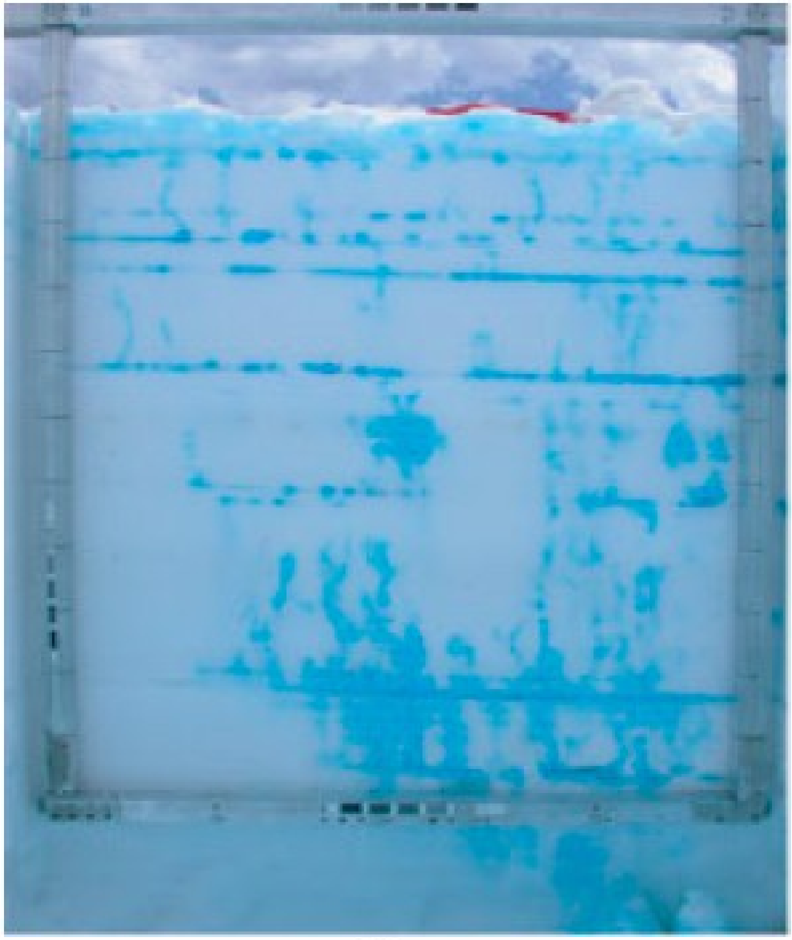
The problem …

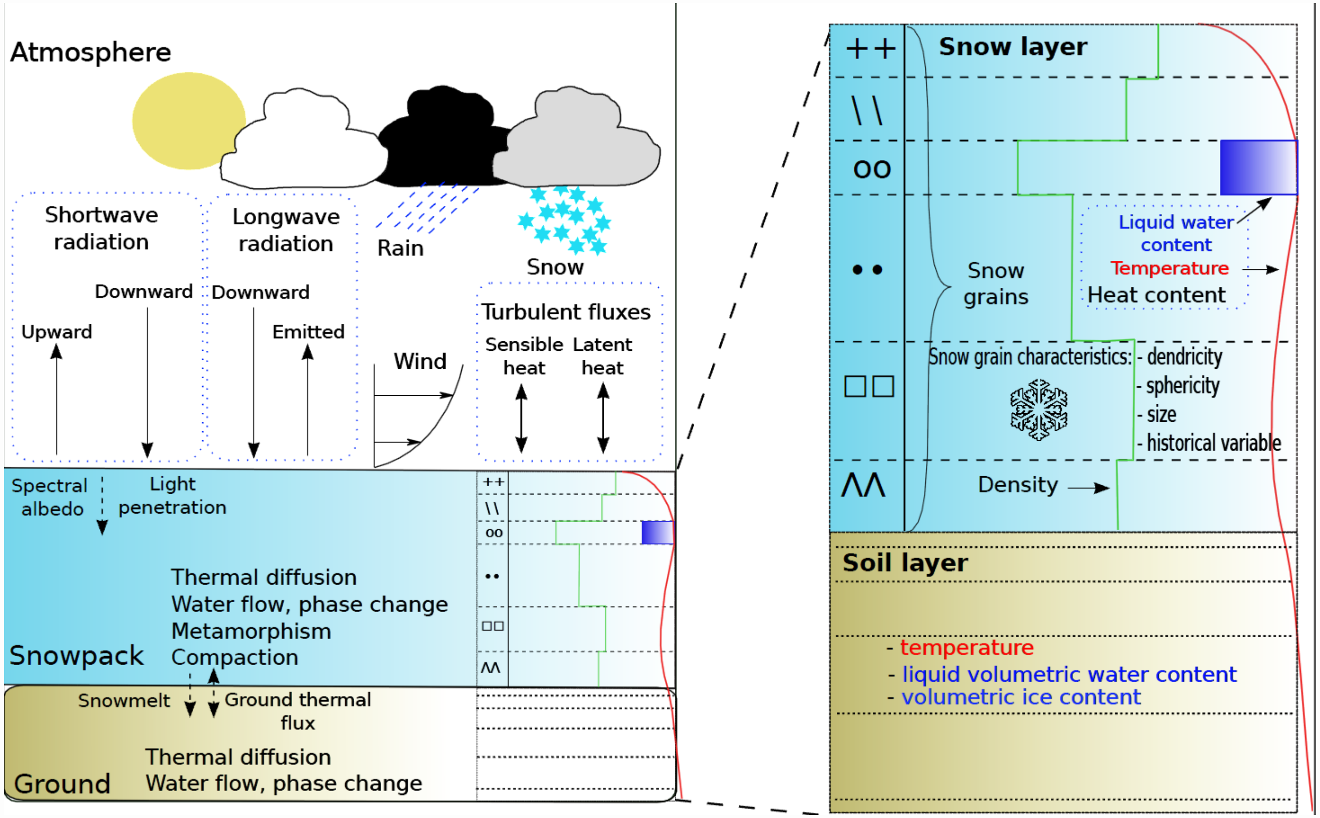




Williams et al.

… its representation in the Crocus snowpack model

- Model overview



Account for most major processes in snow, with a larger number of layers (up to 50+)

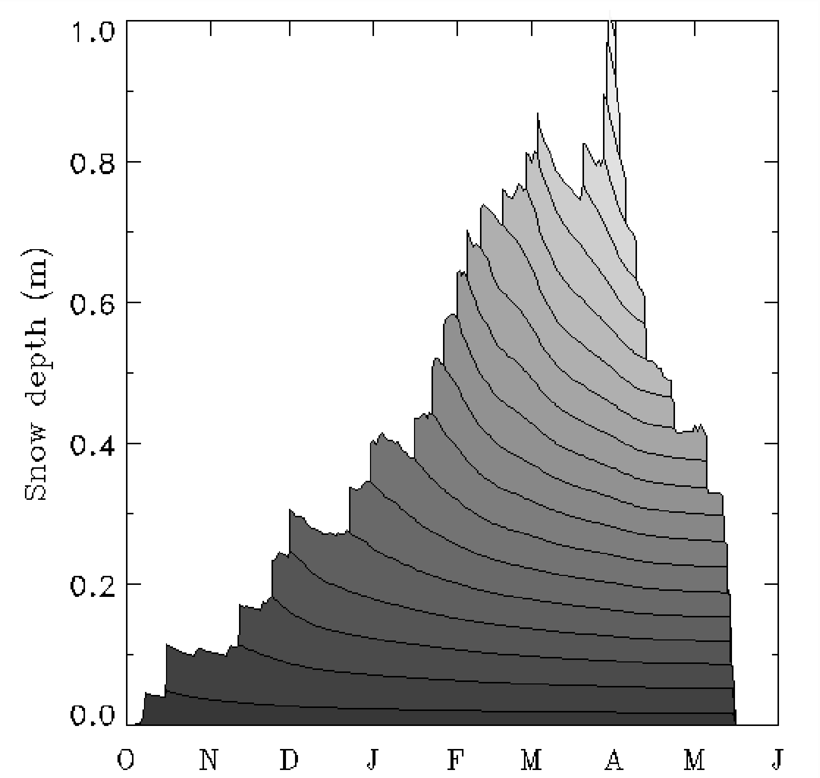
Shares many similarities with SNOWPACK

Generally used for specific applications (avalanche warning, glacier mass balance etc.).

Prognostic variables in snow layers : snow mass (SWE), density (dry + LWC), enthalpy (temperature + liquid water content), specific surface area, other microstructure terms (sphericity, history of layer, layer age)

Crocus snowpack model

- Lagrangian representation of vertical profile : layer thickness changes in time (+ layer merge/split)



from Essery, 2013

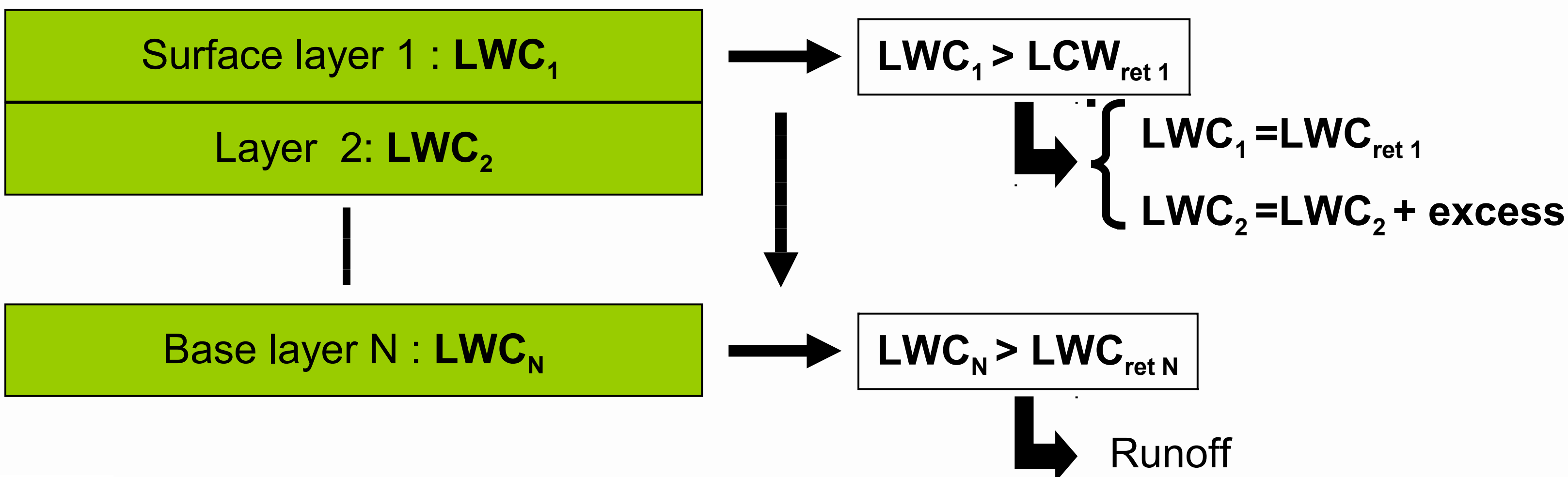
Bucket percolation scheme

Bucket approach : only downwards water movement

- In each layer, add liquid water content due to melt : $LWC = LWC_{init}$ + melt (or substract refreeze)
- Percolation in the snowpack
  - Downwards flow if $LWC > LWC_{retention}$
  - $LWC_{retention}$ = typically 5% of pore volume (Crocus) or functi[on] of density (SNOWPACK) (water retention curves)

$$\theta_h = \begin{cases} 0.0264 + 0.0099\frac{(1-\theta_i)}{\theta_i}, & \theta_i \le 0.23 \\ 0.08 - 0.1023\,(\theta_i - 0.03), & 0.23 < \theta_i \le 0.812 \\ 0, & \theta_i > 0.812 \end{cases} \quad (1)$$

where θ_i is the volumetric ice content of the snow ($m^3\,m^{-3}$).



Bucket percolation scheme

Bucket approach : only downwards water movement
Not too bad for runoff timing at base of the snowpack at daily time scale : Brun et al., 1989 JoG, confirmed by Wever et al., 2013 TC.

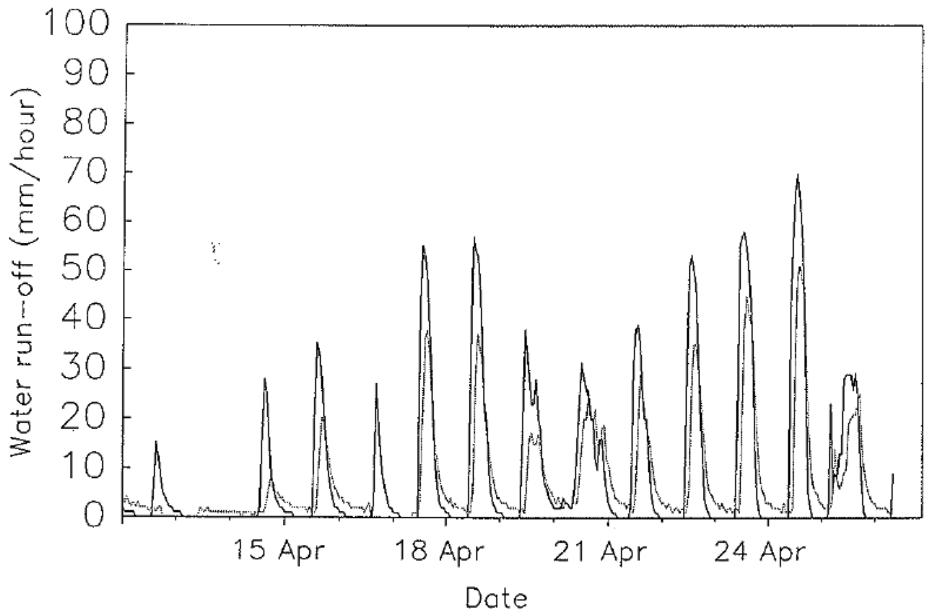
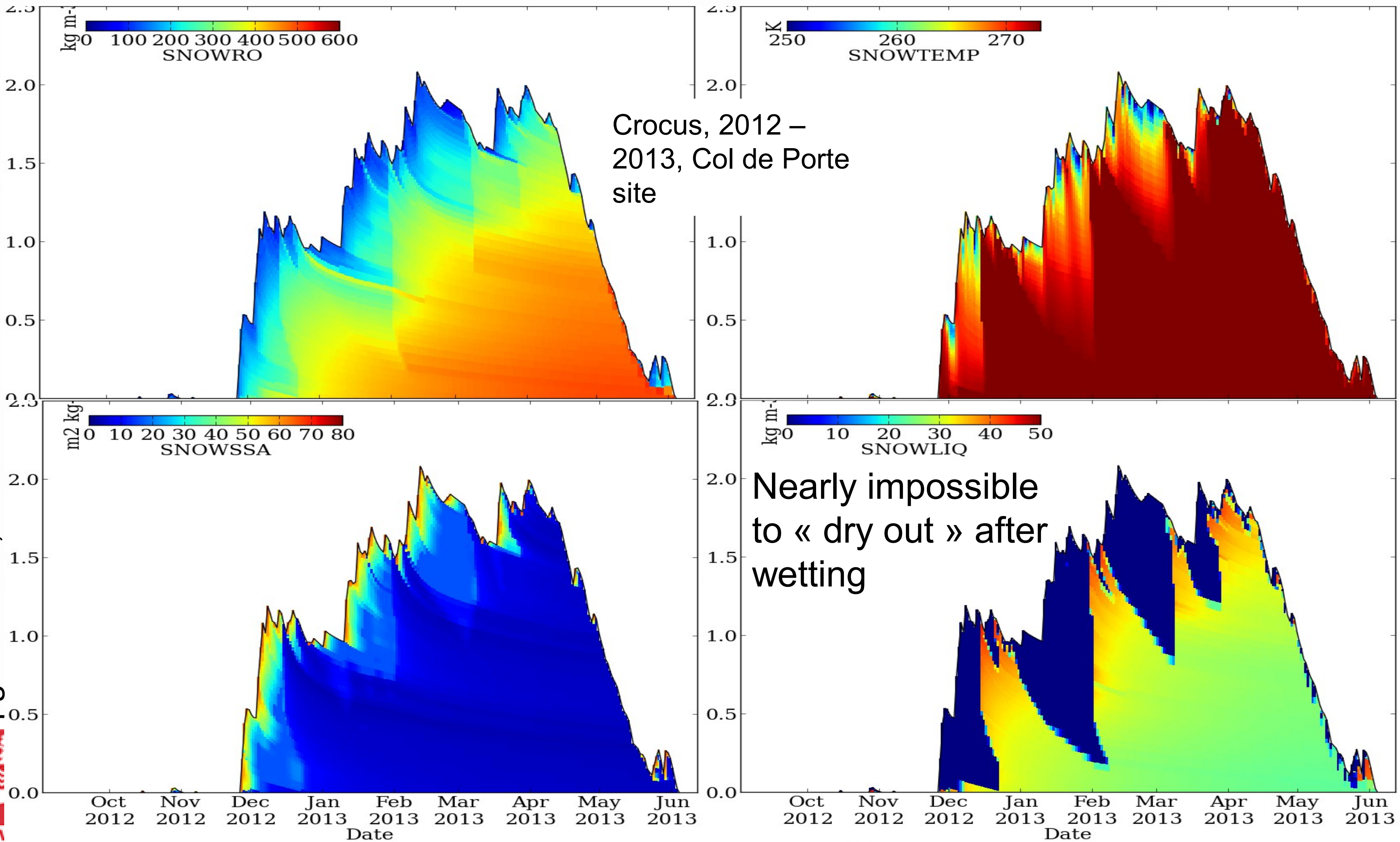


Fig. 9. Measured (--) and simulated (−) water run-off during the third test period at Col de Porte.

Cannot nicely handle refreeze, melt crusts, capillary barriers/rise etc.

Crocus snowpack model

- Example of output



Carmapgnola et al., TC 2013

Richards equation

Generalised Darcy law

V = velocity or volumetric flux

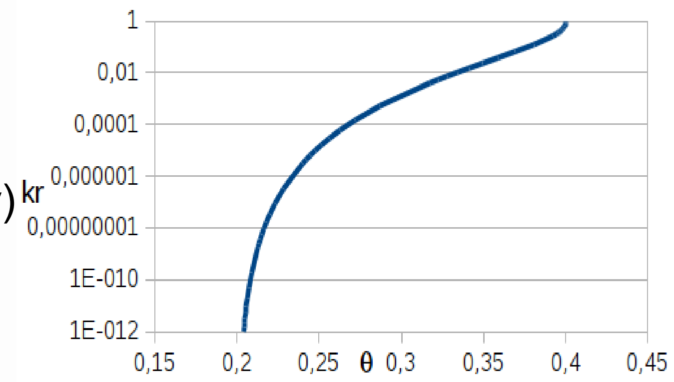
$$\psi = \frac{p}{\gamma} = \frac{p}{\rho\, g}$$

V = $K(\theta)\ (\partial\psi/\partial z + 1)$ where ψ is the pressure head (in m)

→ saturated condition : $\psi > 0$

→ unsaturated condition : $\psi < 0$ (suction = effect of capilarity)

« 1 » = term due to gravity (may be replaced by cos(slope))



Hydraulic conductivity in unsaturated conditions

$K(\theta) = K_{sat} \,.\, Kr(\theta)$

with the **saturated hydraulic conductivity** : $K_{sat} = \rho_w\, g / \mu * K'$

where ρ_w is liquid water density, g is gravitational acceleration, μ is liquid water dynamic viscosity and K' is the intrinsic permeability, which depends on density and SSA (e.g. Shimizu, 1970, Calonne et al., TC 2012)

And kr(θ) is the **relative permeability** = empirical function ∈ [0, 1]

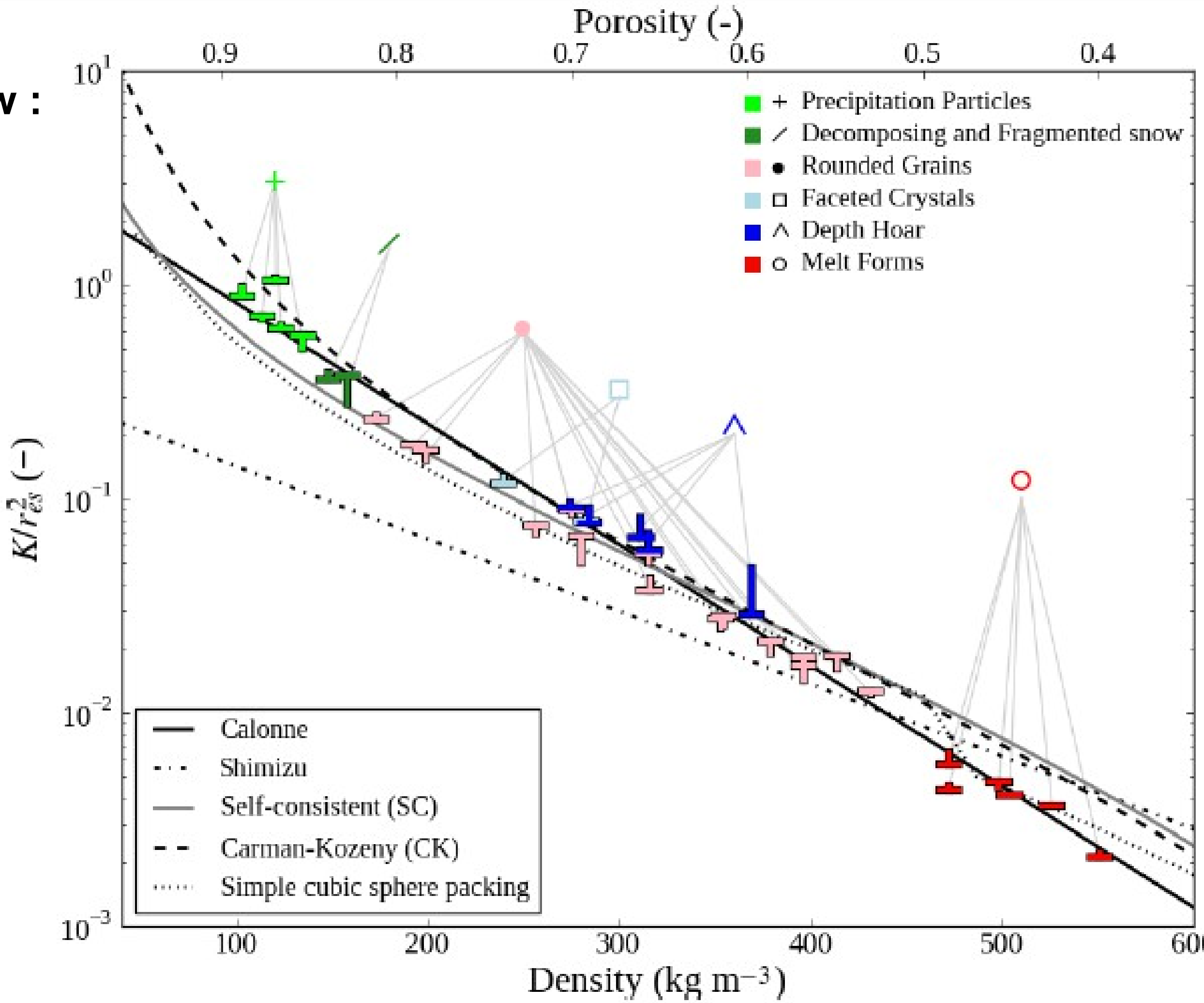
→ dry medium: kr(0) = 0

→ saturated medium: kr(n) = 1

→ **Strong non linearity**

Richards equation

Intrinsic permeability of snow :



$K' = (3.0 \pm 0.3)\, r_{es}^2 \exp((-0.0130 \pm 0.0003)\, \rho_{snow})$ where $r_{es} = 3/(\rho_{ice} * SSA)$

Calonne et al., TC 2012

Richards equation

Richards equation = water balance + generalised Darcy law

Solve simultaneously LWC in each layer accounting for capillary rise, capillary barriers (i.e permeability contrasts).

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K(\theta)\left(\frac{\partial \psi}{\partial z} + 1\right)\right]$$

where

K is the hydraulic conductivity,

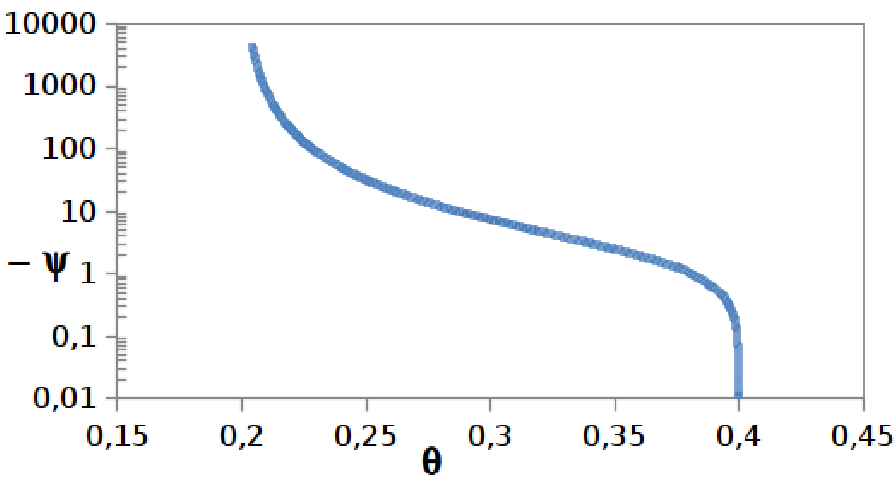
ψ is the pressure head,

z is the elevation above a vertical datum,

θ is the water content, and

t is time.

θ and ψ related by empirical curves (van Genuchten etc.)



Implemented in SNOWPACK by Wever et al. (TC 2013) ; work in progress to implement in Crocus. Should allow better description of ice lens formation and surface snow wetting, in particular.

Richards equation

Richards equation

3 main approach to solve the equation:

- Mixed form: Fluxes estimate → evolution of θ: Iteration until convergence
- θ form: $\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K(\theta)\left(\frac{\partial \psi}{\partial \theta}\frac{\partial \theta}{\partial z} + 1\right)\right]$

  → Bad handling of unsaturated / saturated transistion
- ψ form: $C_{\psi}(\psi)\frac{\partial \psi}{\partial t} = \frac{\partial}{\partial z}\left[K(\psi)\left(\frac{\partial \psi}{\partial z} + 1\right)\right]$

Closed form on ψ using the capillary capacity $C_{\psi}(\psi) = \frac{\partial \theta}{\partial \psi}$

= analytic derivative of the retention curve

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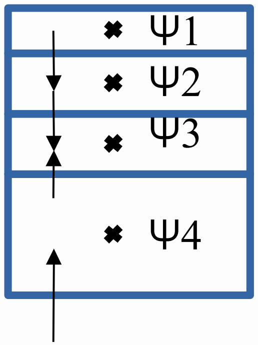
Richards equation

Richards equation : solving strategy



- Finite volumes discretization:

- Ψ = computed at the layer center
- V = computed at the layer boundary

Interfacial permeability at interface between layer a and b with thicknesses Δz_a and Δz_b :

$K_{eq} = K_a K_b * (\Delta z_a + \Delta z_b)/(K_a \Delta z_b + K_b \Delta z_a)$ *[harmonic average]*

(runs into trouble in dry/wet transition zone ... because Θ_{dry} =0 …)

- Dealing with the non-linearity: Picard fixed point loop

- **Outer loop = time steps** (~ 1mn with time step automated evolution)
- **Inner loop = non-linear solver** Picard, solving the tri-diagonal system (up to 15 iterations before time step decrease)

Richards equation

Richards equation : Peculiarity of the snowpack problem

- **Dealing with very dry conditions:** $\psi \rightarrow -\infty$ when $\theta \rightarrow 0$

$kr(\theta) \rightarrow 0$ when $\theta \rightarrow 0$: no flux possible !

Badly defined problem: $V = Ks\, kr(\theta)\, \nabla\psi$

small big

→Empirical solution: creating an artificial small amount of water but it may require a correction of the final mass balance...

- **Potential presence of stiff change on the $\Psi(\theta)$ and $kr(\theta)$ between two cells**

The boundary between fresh and old snow may present a drastic change of hydraulic properties due to snow metamorphism → Potential source of numerical troubles...

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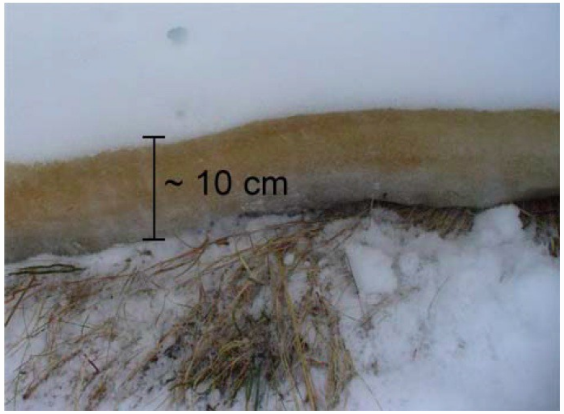
Coupling to atmosphere

- Rain on snow / surface melt induce increase in liquid water content of uppermost snow layer
- Evaporation induces decrease of liquid water content of uppermost snow layer
- Both are accounted for in Crocus already

Coupling to the ground

- ISBA multi layer heat diffusion / Richards equation scheme in the soil (accounting for phase change)
- Coupling between ISBA ground and Crocus at each time step
- In bucket model, snow runoff drips into the ground (which partitions into runoff/infiltration depending on saturation level in top level of ground) : no feedback of soil moisture content into Crocus
- In Richards equation, soil moisture needs to feed in Crocus (because of potential ground water capillary rise in snow – see Coleou and Lesaffre, 2001, picture below from Mitterer et al., 2012)



- Numerical stability issue (?): The soft coupling between ISBA and Crocus may create problems...

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Perspectives

- Still issues with θ_r definition, the fact that snow can be really entirely dry at time (in contrast to other environmental matrices) -> mass conservation issues in the code.
- Chris will be back 1 month the coming summer !


Thank you
for your attention
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