On the role of the bed-load thickness in the dune instability mechanisms

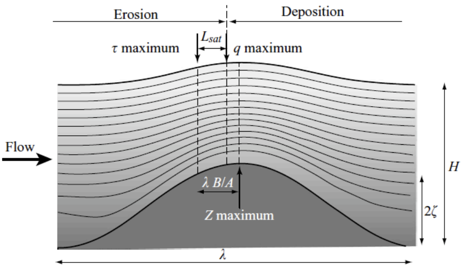
and how numerical simulation could help ...

Julien Chauchat[1]

[1] LEGI UMR 5519 - INPG / ENSE3 (julien.chauchat@grenoble.inp.fr)

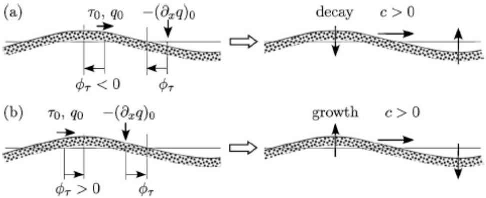
Atelier Fluide OSUG

18^{th} june 2012



Fourriere et al., JFM (2010)

Charru, PoF (2006)



Destabilising mechanisms:

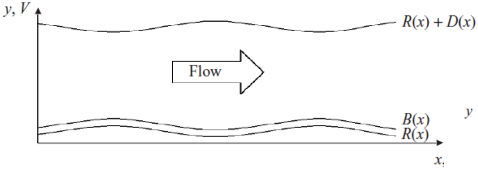
→ Phase shift between the bed shear stress and the topography due to fluid inertia
eg. Kennedy, JFM (1963) ; Ouriemi et al. JFM (2009)

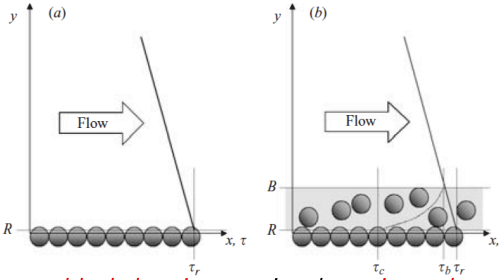
Stabilising mechanisms:

→ Gravity *e.g. Kennedy, JFM (1963)*
→ Bed-load thickness *Colombini, JFM (2004)*
→ Saturation length *Fourriere et al., JFM (2010)*
→ Free surface *Fourriere et al., JFM (2010)*
→ Suspended load *Engelund and Fredsoe,* ARFM (1974)

⇒ No clear picture of bedforms formation !!!

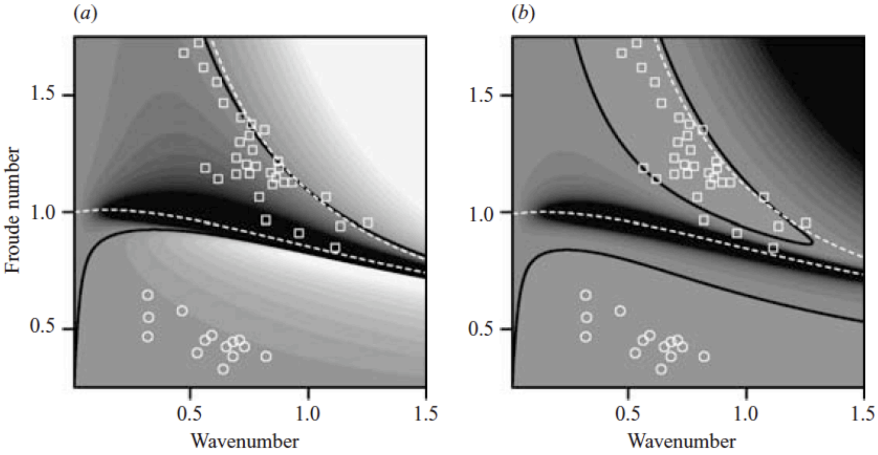
Colombini, JFM (2004)





... the phaselag between sediment transport and bed elevation remains the main mechanism driving instability. However, it is shown that this phase-lag varies significantly in a neighbourhood of the bed. Moreover, since the layer in which sediments are moving has a finite (though small) thickness, it is assumed that the perturbations of the fluid stress driving bedload transport should be evaluated at the top of the layer itself. It is shown that such an apparently minor modification of the classical approach alters remarkably the balance between stabilizing and destabilizing effects that drives the instability process.

Colombini, JFM (2004)



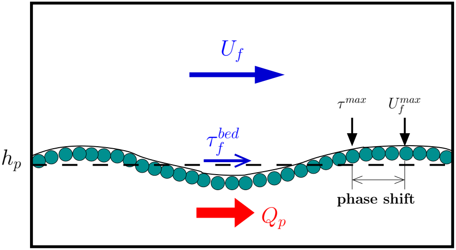
Bed-load thickness = 0 **Empirical bed-load thickness**

Growth rate plot ; experiments of Guy et al. (1966): ○ dunes; □, antidunes.

Dark color = Stable ; Thick solid lines = marginal curves

Colombini, JFM (2004)

Particle Flux



Navier-Stokes

+

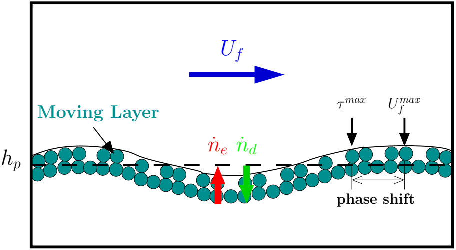
Mass conservation

$$\phi \frac{\partial h_p}{\partial t} + \frac{\partial Q_p(\theta)}{\partial x} = 0$$

$\rightarrow$ $\theta = \frac{\tau_f^{bed}}{\Delta \rho g d}$: Shields Number

[Engelund and Fredsoe, 1982, Richards, 1980, Colombini, 2004],...

Erosion / Deposition



Navier-Stokes

+

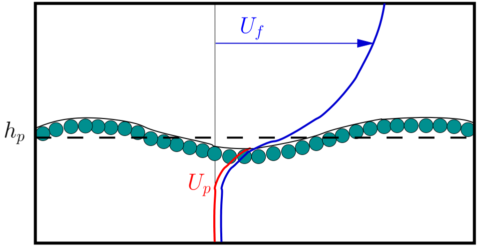
Mobile particle number conservation

$$\frac{\partial n}{\partial t} = \dot{n}_e - \dot{n}_d - \frac{\partial nu}{\partial x}$$

$\rightarrow$ $\dot{n}_e$, $\dot{n}_d$: Erosion / Deposition rate

[Charru et al., 2004, Charru, 2006], ...

Two-phase model for bed-load transport in laminar flows



Navier-Stokes

$+$

$\rightarrow$ *Mass conservation equation*

$\rightarrow$ *Momentum equation*

Fluid + Particles

Closures :

- Fluid: Newtonian rheology - Einstein's viscosity $\rightarrow \eta_e = \eta_f \ (1 + 5/2 \ \phi)$
- Particles: Frictionnal rheology - Coulomb or $\mu(I) \rightarrow \tau^p = \mu \ p^p$
- Fluid-particles interaction : Buoyancy + Darcy $\rightarrow \phi \overrightarrow{\overrightarrow{\nabla}} \tau^f + \frac{\eta_f \epsilon^2}{K} (\overrightarrow{u^f} - \overrightarrow{u^p})$

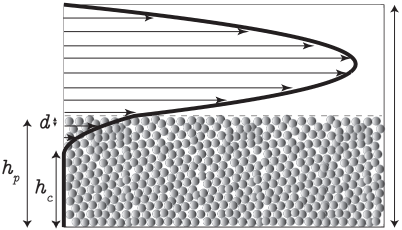
Set of equations to be solved

$\rightarrow$ Darcy-Brinkman for the fluid (Darcy dominant)

$\rightarrow$ Mixture momentum equation (Fluid+Particles)

Ouriemi, Aussillous and Guazzelli, JFM (2009)

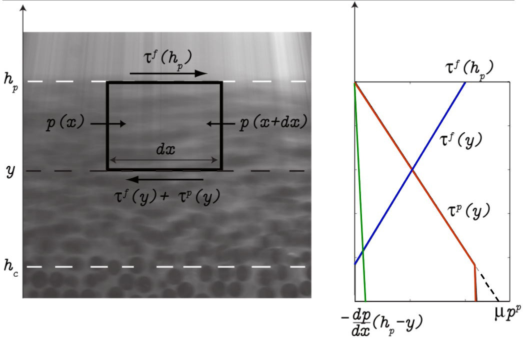
Analytical solution (Coulomb)



Ouriemi et al. (2009)

Mixture momentum balance

$$\tau^m(y) = \tau^p(y) + \tau^f(y) = \tau^f(h_p) - \frac{\partial p^f}{\partial x}(h_p - y)$$



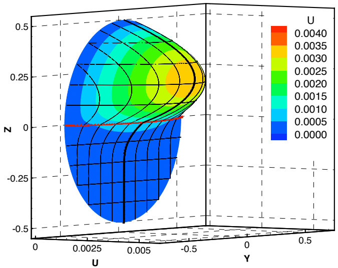
Particle shear stress: Coulomb $\tau^p \leq \mu_s p^p = \mu_s \phi_0 \Delta\rho g (h_p - y)$ where p^p is hydrostatic

Fluid shear stress: $\tau^f = \eta_e \dfrac{\partial u^f}{\partial y}$

Velocity profile: $u^p \approx u^f \approx \dfrac{(\mu\phi_0\Delta\rho g + \frac{\partial p^f}{\partial x})}{\eta_e} \dfrac{(y - h_c)^2}{2} \Rightarrow q_p$ and $q_f \propto \boxed{q_0 = \Delta\rho g h_f^3/\eta}$

Eulerian model → **No influence of the particle size !** Shields number θ not the right parameter

3D Finite Element Model



Initial numerical model (Médale)

- 3D Navier-Stokes equations
- Finite Element Method
  - Velocity: Quadratic elements
  - Pressure: Linear elements
- Newton-Raphson algorithm

Non-dimensionalisation [Ouriemi et al., 2009]

Length: H ; Pressure: $\Delta\rho\ g\ H$; Time: $\eta_f\ /\ \Delta\rho\ g\ H$

3D two-phase numerical model (Chauchat & Médale, 2010)

1. Mixed-fluid model: mixture momentum equation $\Rightarrow$ single effective phase
2. Two-fluid model: fluid and particle momentum equations

Issues

- Modeling friction (regularization)
- Coupled problem (fluid-particle interaction)

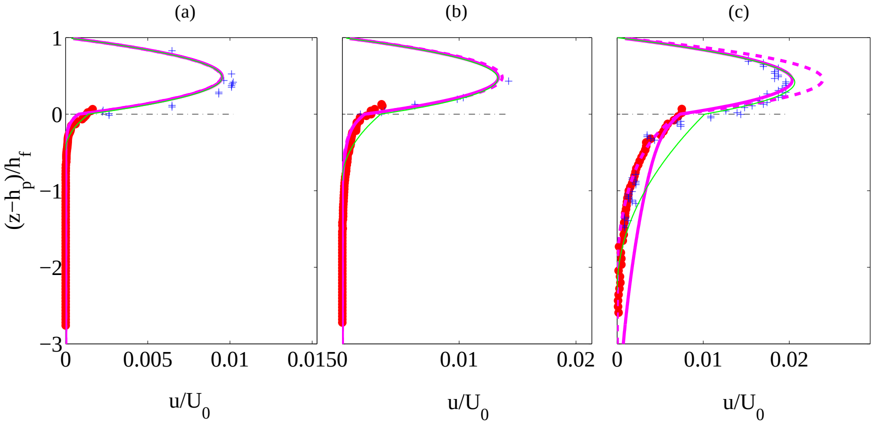
$U_0 = \Delta\rho g h_f^2/\eta_f$

— Coulomb 2D, — $\mu(I)$ 2D, -- $\mu(I)$ 3D

Borosilicate (d=1 mm) $Q_f = 3.58\,10^{-6}$ m^3/s

Borosilicate (d=1 mm) $Q_f = 5.25\,10^{-6}$ m^3/s

PMMA (d=2 mm) $Q_f = 1.23\,10^{-6}$ m^3/s



→ **3D two-phase numerical model (Chauchat & Médale, CMAME 2010)**

Aussillous *et al.*, submitted to JFM

Conclusion

→ Continuous model for bed-load transport

→ Phenomenological rheology used to describe intergranular stresses $\mu(I)/\phi(I)$

→ 3D numerical model

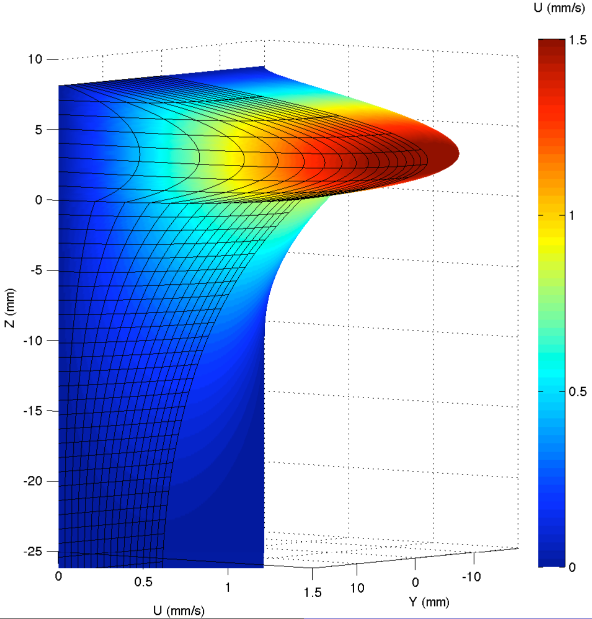
Perspectives

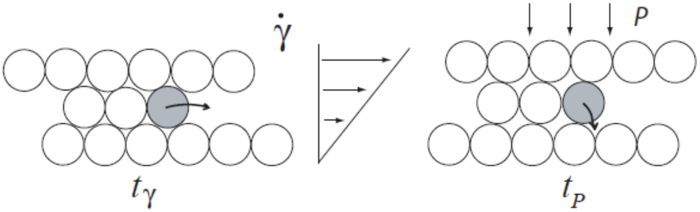
→ Study numerically bedforms formation

Collaborations

- IUSTI (Marseille) : P. Aussillous, E. Guazzelli, M. Médale
- LEGI (Grenoble) : E. Barthélémy, D. Hurther, H. Michallet, T. Revil-Baudard
- Univ. Savoie (Chambéry) : M. Pailha

3D velocity profile





Andreotti, Forterre and Pouliquen (2011)

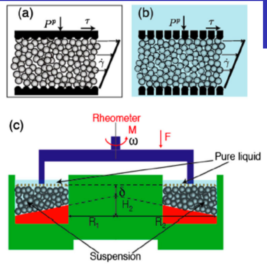
Interpretation of the Viscous number:

$$I_v = \frac{\dot{\gamma}\ \eta_f}{\alpha\ p^p} = \frac{t_{micro}}{t_{macro}} \quad where \quad t_{micro} = \frac{\eta_f}{\alpha\ p^p} \ \ and \ \ t_{macro} = \frac{1}{\dot{\gamma}}$$

Cassar, Nicolas and Pouliquen (2005), Boyer, Guazzelli and Pouliquen (2011)

The shear stress is proportional to the pressure and depends on I_v, also the volume fraction depends on I_v:

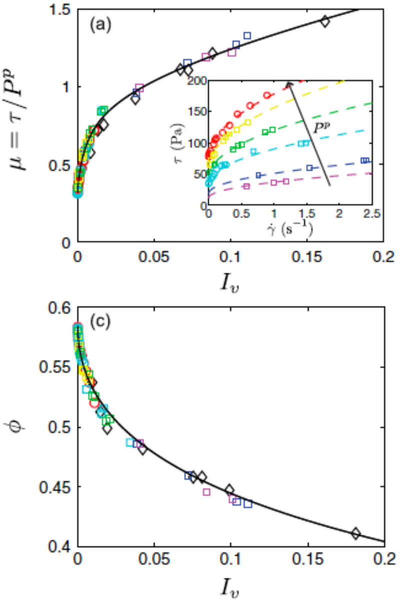
$$\boxed{\tau = \mu(I_v)\ p^p \quad and \quad \phi = \phi(I_v)}$$



Two contributions:

$$\mu(I_v) = \mu^c + \mu^h$$

$\mu^c \rightarrow$ contact
$\mu^h \rightarrow$ hydrodynamic



Effective viscosity:

$$\mu^h(I_v) \Rightarrow \frac{\eta_e(\phi)}{\eta_f} = 1 + \frac{5}{2}\phi\left(1 - \frac{\phi}{\phi_m}\right)^{-1}$$

Friction coefficient:

$$\mu^c(I_v) = \mu_s + \frac{\mu_2 - \mu_s}{I_0/I_v + 1}$$

Volume fraction:

$$\phi(I_v) = \frac{\phi_m}{1 + I_v^{1/2}}$$

where $\phi_m = 0.585$, $\mu_s = 0.32$, $\mu_2 = 0.7$ and $I_0 = 0.005$

Boyer, Guazzelli and Pouliquen (2011)

[Charru, 2006] Charru, F. (2006).
Selection of the ripple length on a granular bed sheared by a liquid flow.
Physics of Fluids, 18(12):121508.

[Charru et al., 2004] Charru, F., Mouilleron-Arnould, H., and Eiff, O. (2004).
Erosion and deposition of particles on a bed sheared by a viscous flow.
Journal of Fluid Mechanics, 519(-1):55–80.

[Colombini, 2004] Colombini, M. (2004).
Revisiting the linear theory of sand dune formation.
Journal of Fluid Mechanics, 502(-1):1–16.

[Engelund and Fredsoe, 1982] Engelund, F. and Fredsoe, J. (1982).
Sediment ripples and dunes.
Annual Review of Fluid Mechanics, 14(1):13–37.

[Richards, 1980] Richards, K. J. (1980).
The formation of ripples and dunes on an erodible bed.
Journal of Fluid Mechanics Digital Archive, 99(03):597–618.