STABILITE ET EQUILIBRE STATISTIQUE

Joël Sommeria

CNRS-LEGI Grenoble, France

JUPITER
rapid rotation (11 hours)

-shallow layer: 2D turbulence

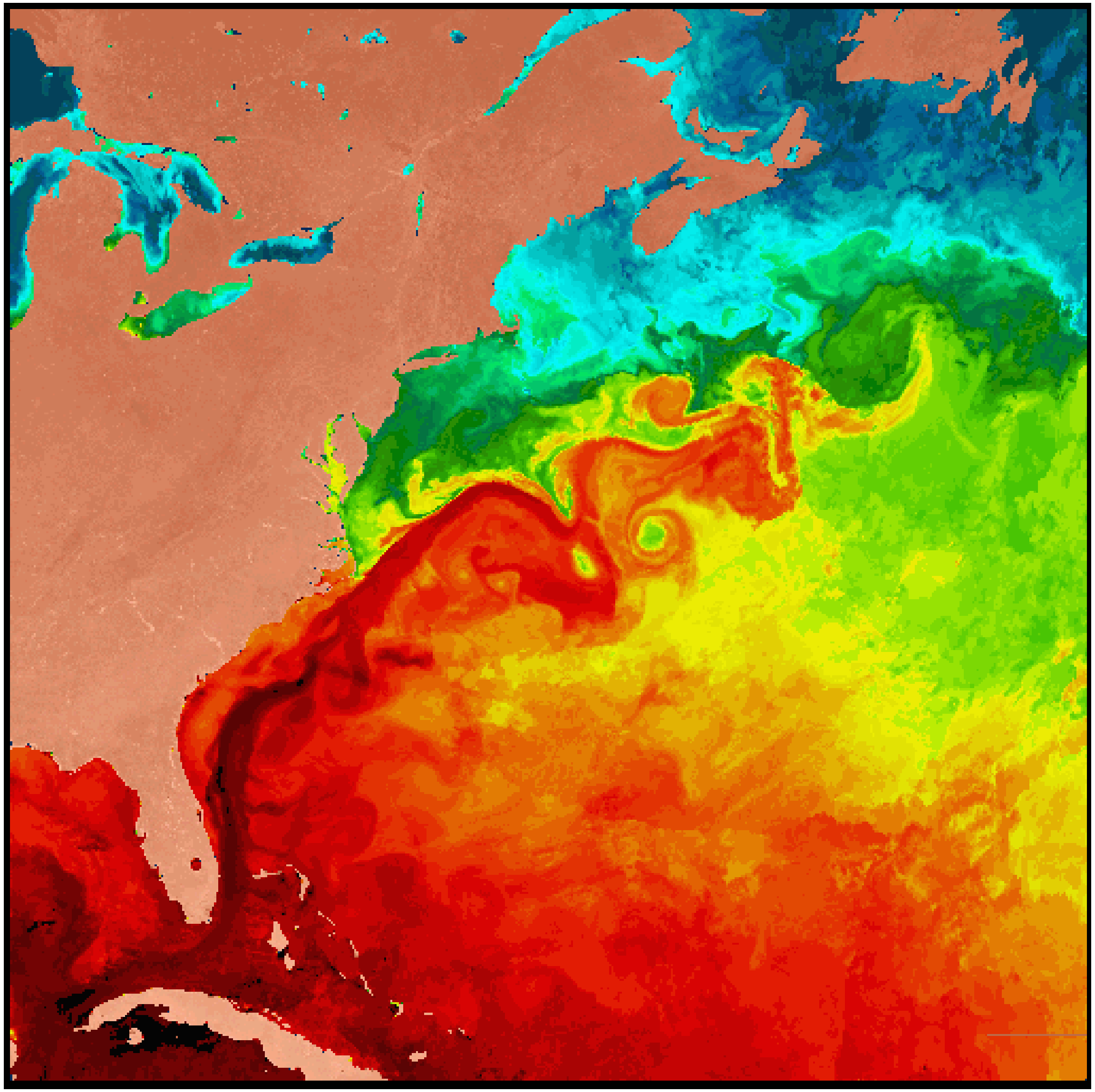
-density stratification + rotation -> geostrophic turbulence (2D or layerwise 3D)

* Potential vorticity conservation

* Energy conservation

inertial dynamics

OCEAN CIRCULATION



infra-red image (temperature field)

-coherent jets

-long-lived vortices

-bimodal behaviour Kuroshio

But large scales quasi-linear: Rossby waves

Statistical mechanics of vorticity

Onsager (1949), Miller(1990), Robert (1990), Robert and Sommeria (1991)

2D Euler equations.

- Conservation of the vorticity for fluid elements (Casimir constants) but extreme filamentation.
- Statistical description by a local pdf: $\rho(\sigma,\mathbf{r})$

  σ vorticity value, $\mathbf{r}$ position, local normalisation $\int\rho(\sigma,\mathbf{r})\, d\sigma=1$

- Maximisation of a mixing entropy: $S=-\int\rho \ln\rho\, d^2\mathbf{r}$ with the constraint of energy conservation
- Energy is purely kinetic but can be expressed in terms of long range interactions:

  Mean field approximation:

  $-\Delta\psi=<\sigma>$ (ψ smooth)

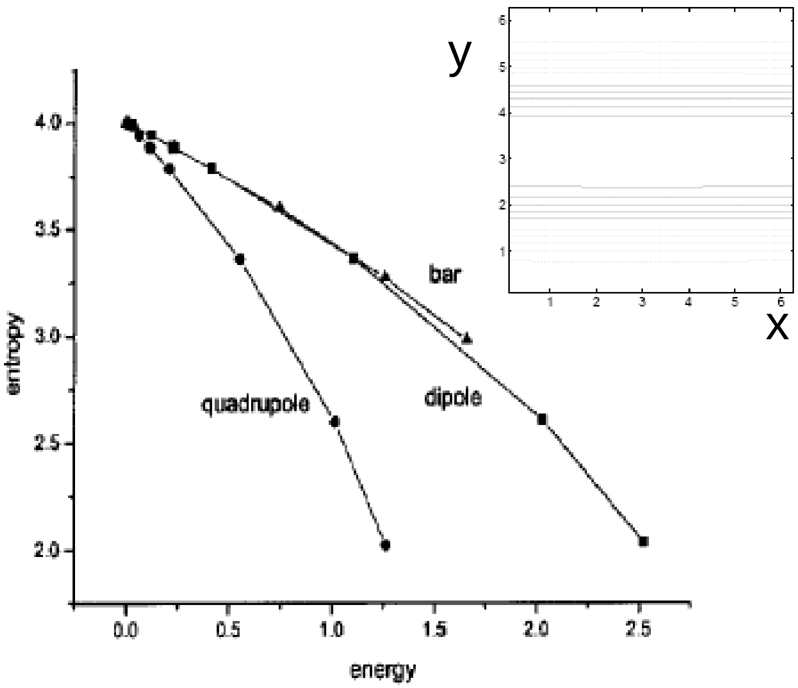
  $E=\int\psi<\sigma>dxdy$, with $<\sigma>=\int\rho(\sigma,\mathbf{r})\,\sigma\, d\sigma$

Dipole vs bar in the doubly-periodic domain

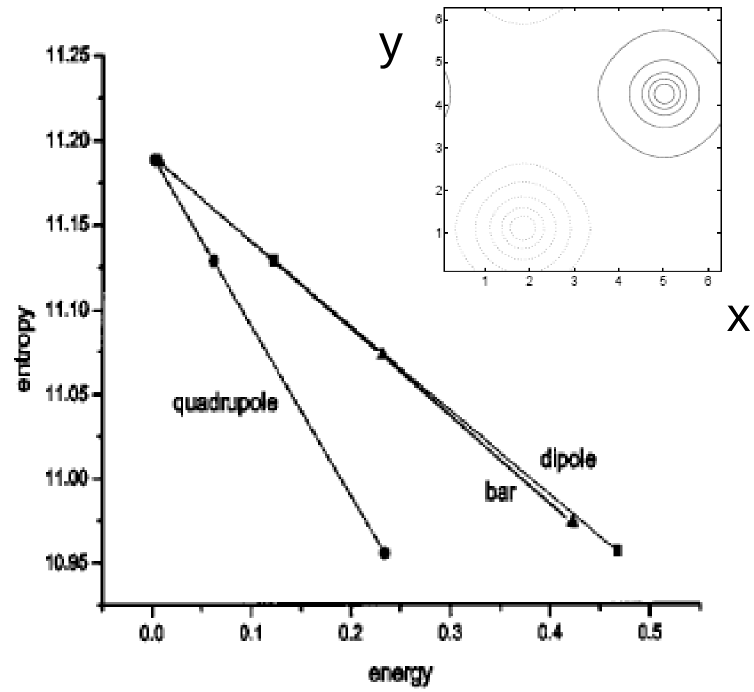
Z. Yin, D.C. Montgomery, and H.J.H. Clercx, Phys. Fluids **15**, 1937-1953 (2003).

Patch vorticity values: 0, +-σ

bar



dipole



Domain area/patch area=3.8

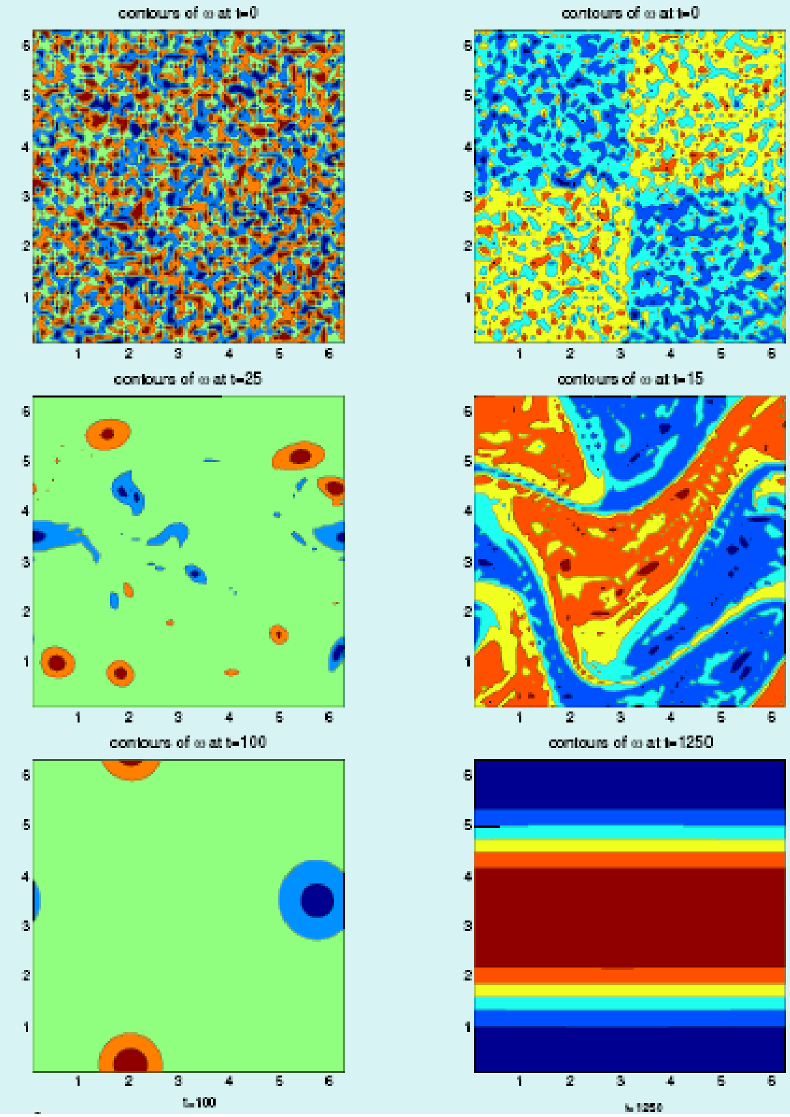
Domain area/patch area=100

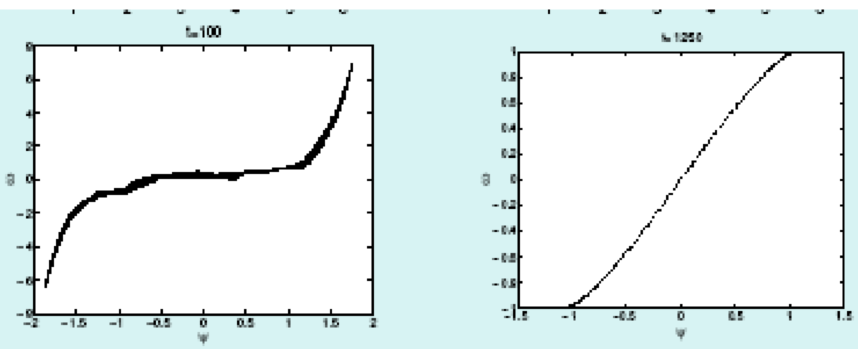
~ point vortices

Numerical test

Z. Yin, D.C. Montgomery, and H.J.H. Clercx
"Alternative statistical-mechanical descriptions of decaying two-dimensional turbulence in terms of 'patches' and 'points'"
Phys. Fluids (2003).

Two numerical simulations of decaying turbulence are carried out with a 2D pseudospectral code which solves the Navier-Stokes equation. For both runs (the left column and the right column): $Re = 5000$; Resolution - 512^2; $\Delta t = 0.0005$.





Not only the final states are predicted by the theories, but also the $\omega - \psi$ plots of the late time correspond to the sinh- and tanh-poission results, respectively.

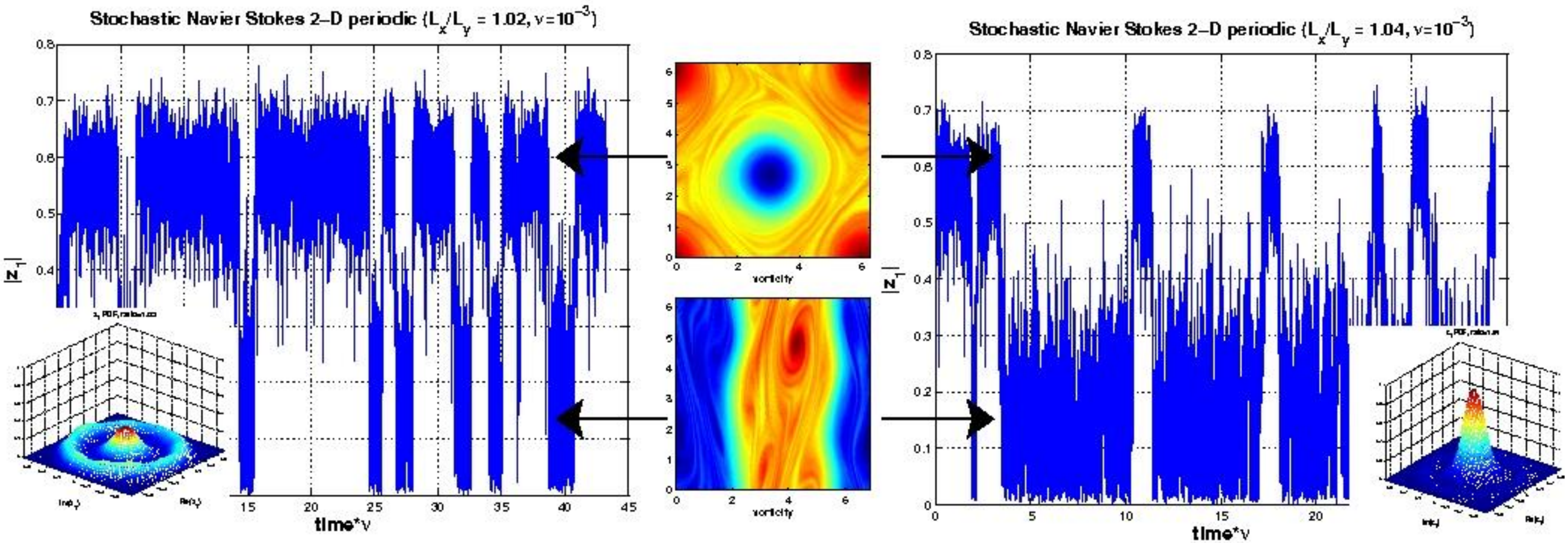
Random changes of flow topology

F. Bouchet and E. Simonet, Phys. Rev. Letters 2008

- 2D turbulence in a periodic domain

- Random forcing of vorticity with white noise

$\delta=1.02$ $\delta=1.04$

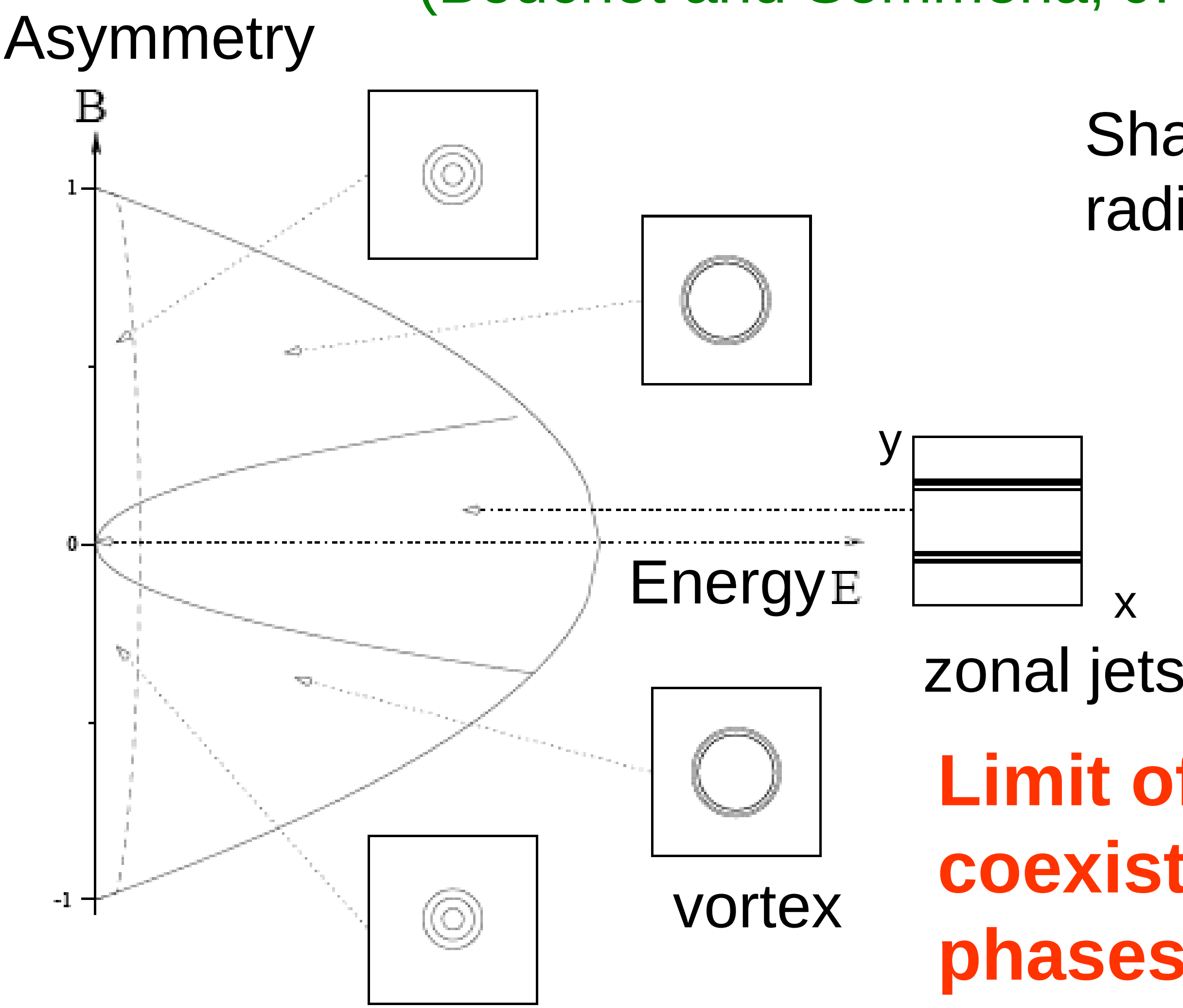


Laboratory experiments: Sommeria(1986), square box

Weeks, Tian, Urbach, Ide, Swinney and Ghil (1997), rotating annulus

Extension to the QG model

(Bouchet and Sommeria, JFM 2002)



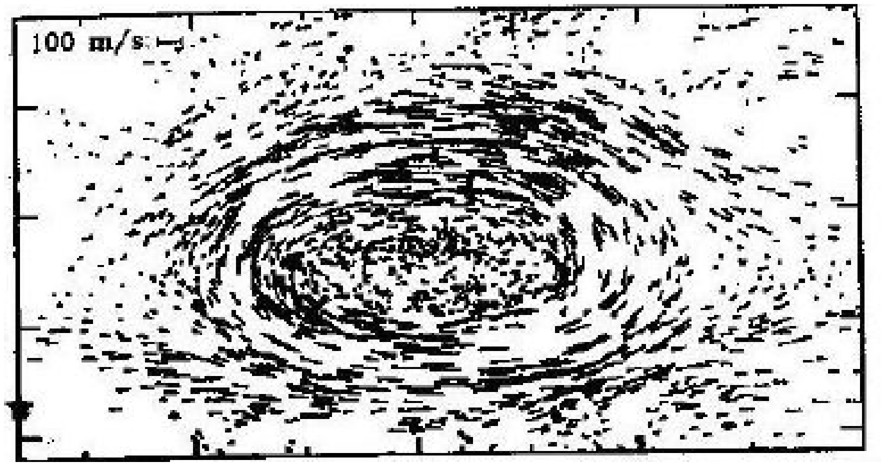
Shallow layer, *R*=Rossby radius of deformation

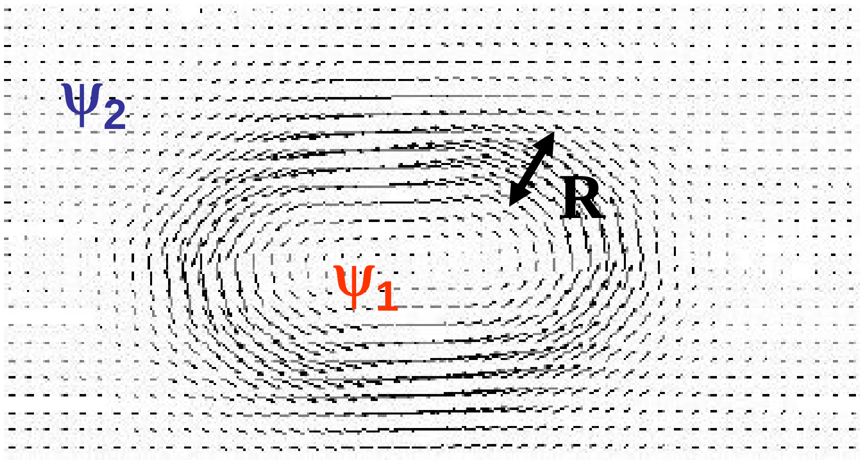
$$q = -\Delta\psi + \psi / R^2$$

Limit of small *R, large E*: coexistence of two phases with uniform PV

(PV staircase)

Application to the Great Red Spot of Jupiter (Bouchet & Sommeria, JFM 2002)

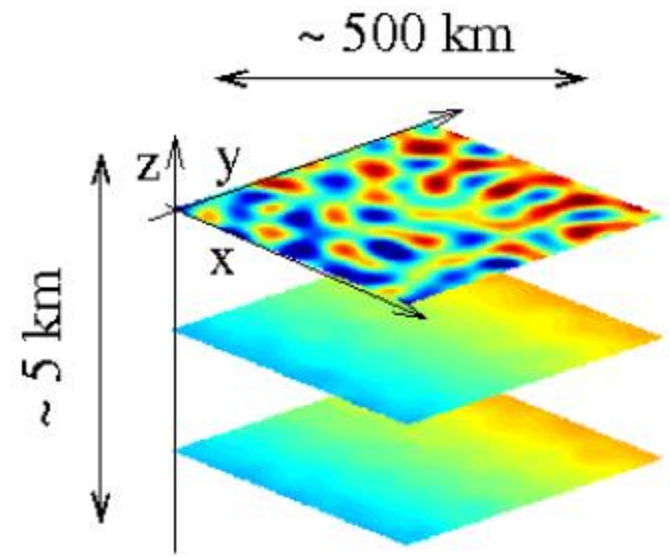


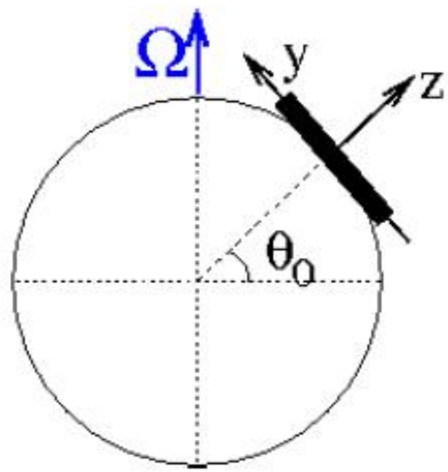


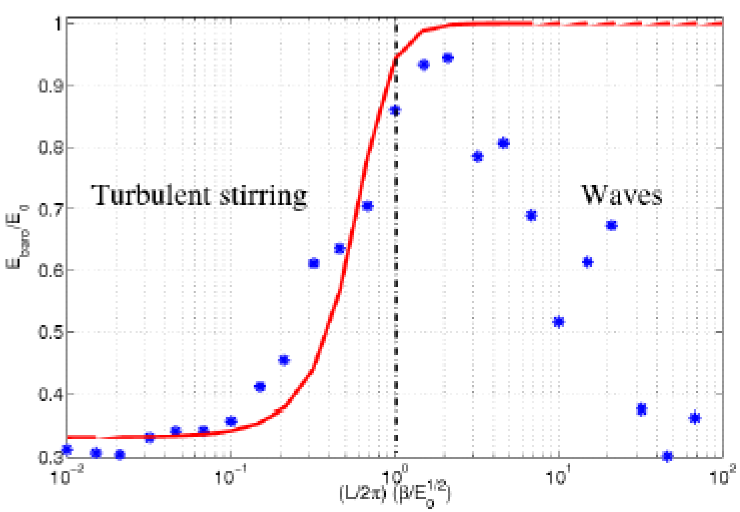
Velocity measured from cloud motion (Dowling and Ingersoll 1989)

Prediction:
- The jet width is of the order of the radius of deformation
- The elongated shape is controlled by the deep zonal shear flow

‘Barotropisation’ de tourbillons océaniques (Venaille Vallis 2012)

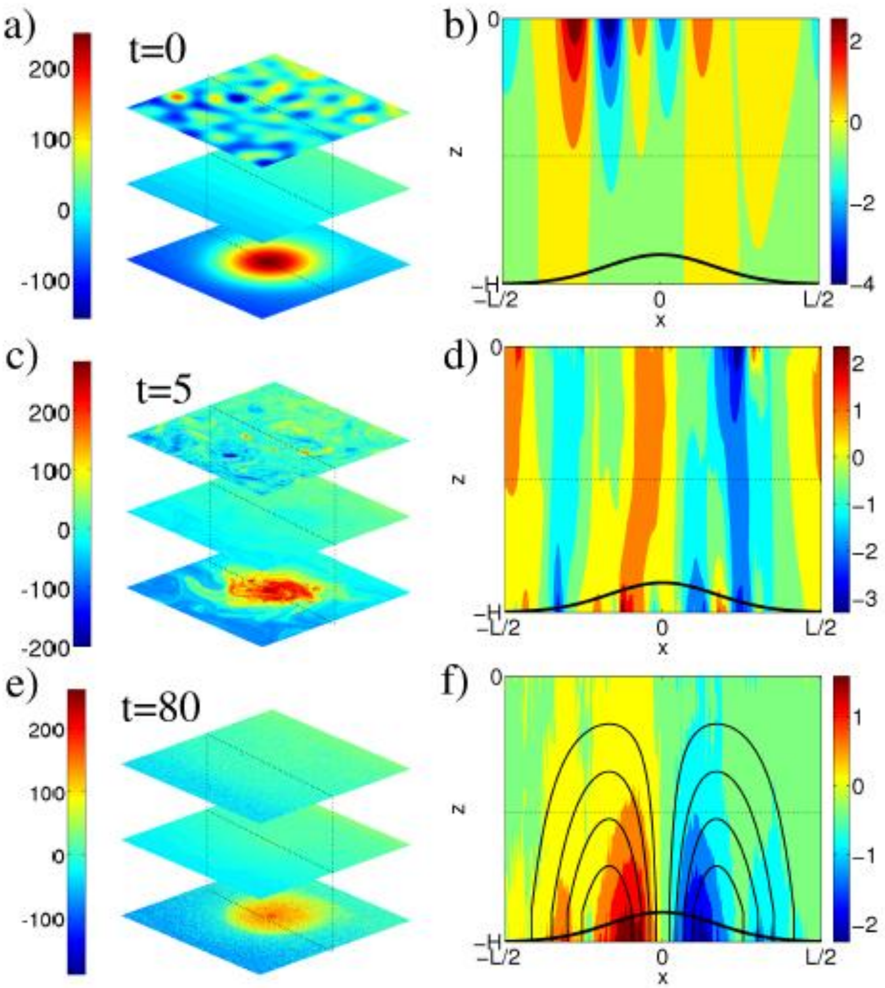






Equilibrium statistical mechanics provides a physical explanation, and quantitative predictions for this phenomenon.

Intensification sur topographie (Venaille Vallis 2012)



Formation of bottom trapped flow along topography contours.

Large-scale organisation of 3D turbulence: von Karman flow

P-P Cortet[1,2], E Herbert[1], A Chiffaudel[1], F Daviaud[1], B Dubrulle[1] and V Padilla[1]

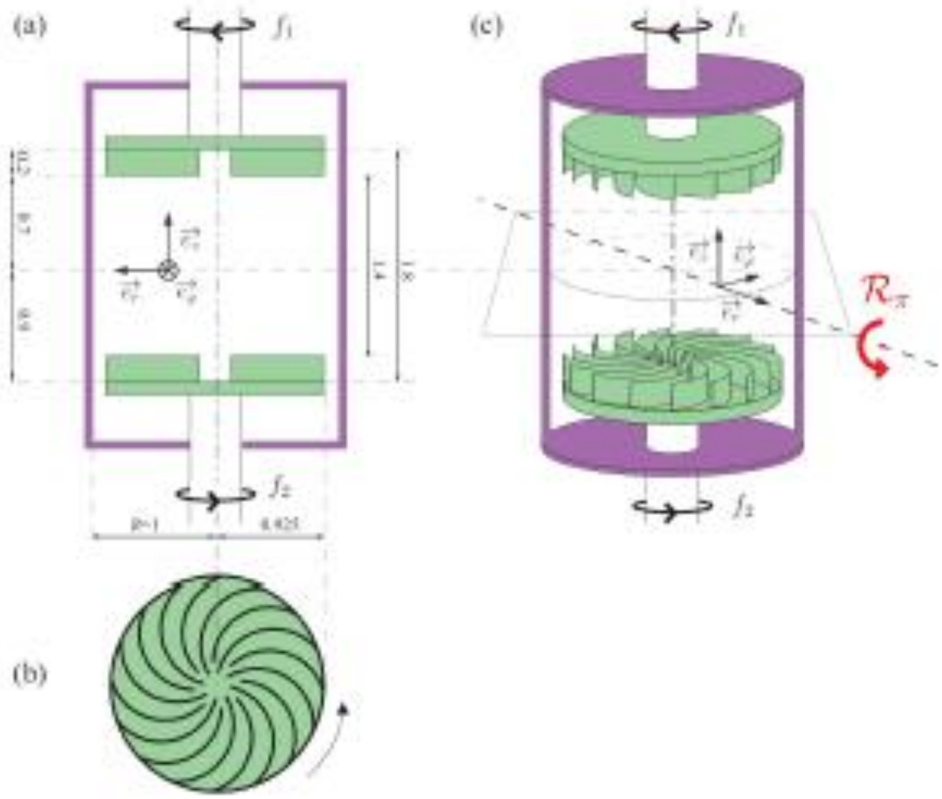


Figure 1. Schematic view of (a) the experimental setup and (b) the impellers blade profile. The arrow indicates the rotation sense. (c) 3D-view and symmetry : the system is symmetric with respect to any $\mathcal{R}_\pi$-rotation of angle π around any line in the equatorial plane which crosses the rotation axis.

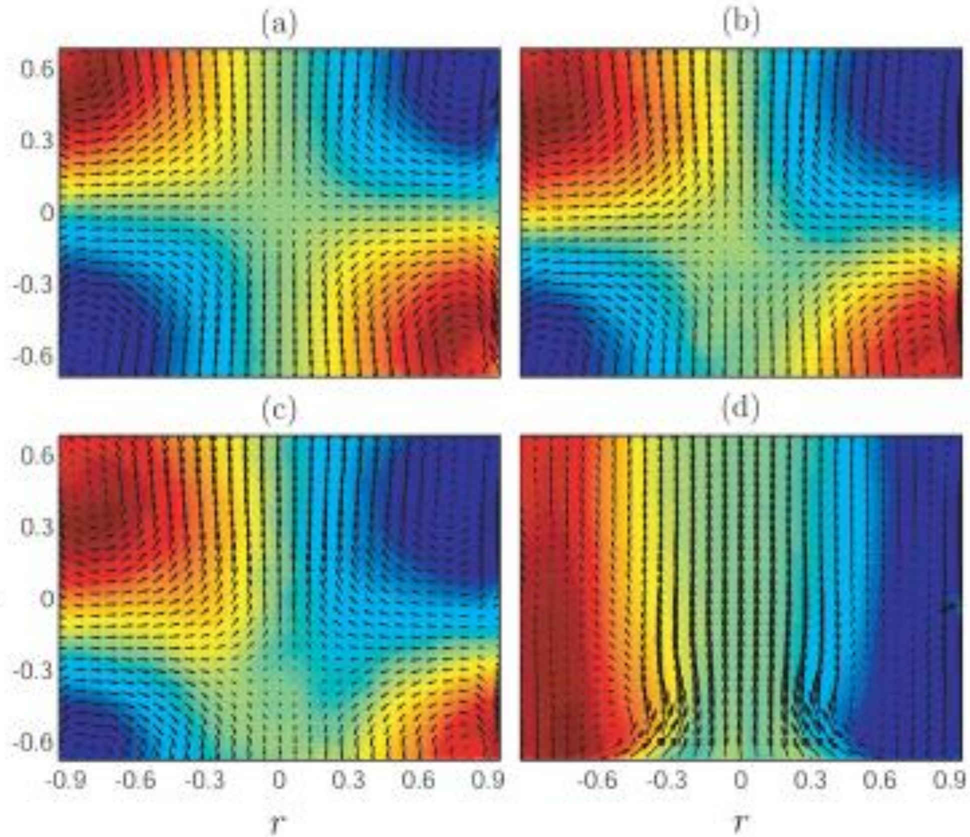
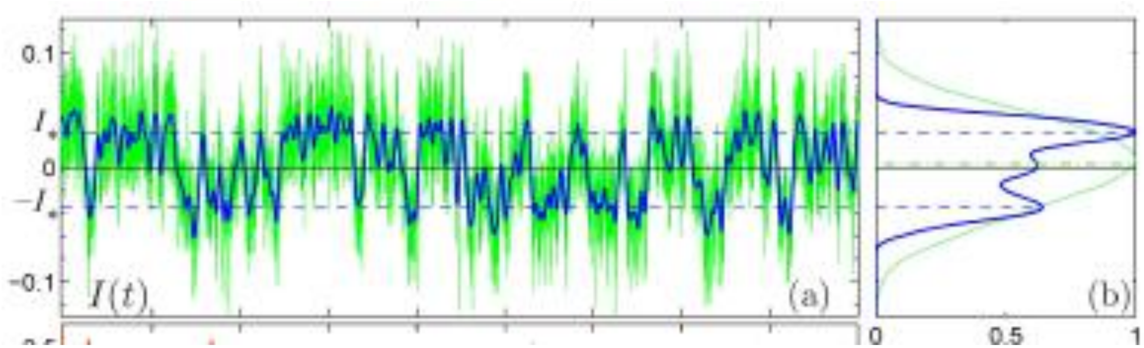


Figure 6. Maps of mean velocity fields of the turbulent von Kármán flow at $Re = 890\,000$ for different values of θ: (a) $\theta = 0$, (b) $\theta = -0.0036$, (c) $\theta = -0.0147$, (d) $\theta = -1$, with same layout as figure 2. The $r \leftrightarrow -r$ symmetry of the maps reveals that the time-averaged mean fields are axisymmetric.



Axisymmetric turbulence statistical mechanics

Aurore Naso[1,2], Simon Thalabard[1,2], Gilles Collette[1,3], Pierre-Henri Chavanis[4], and Bérengère Dubrulle[1]

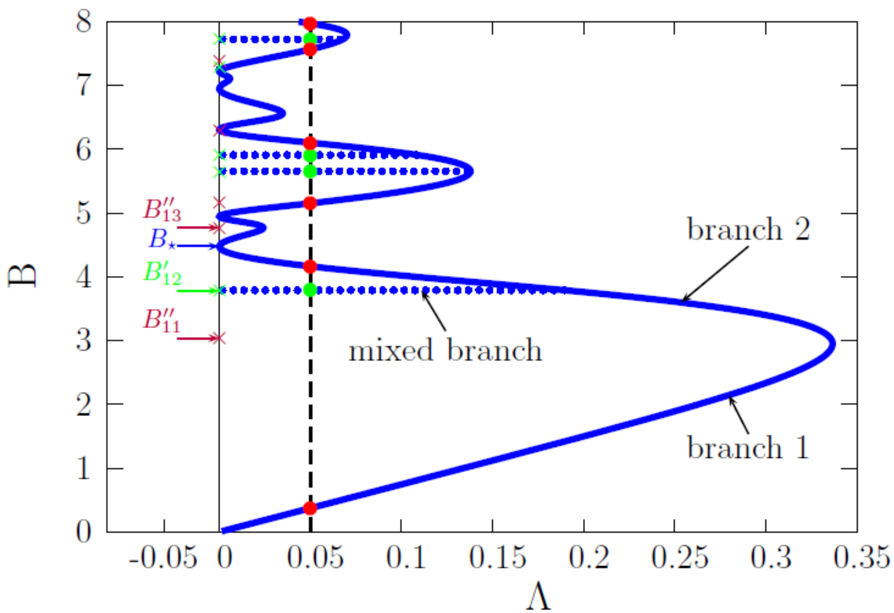


FIG. 1: Top: B as a function of Λ for case L (we have taken $R = 1.4$ and $h = 1.2$). For a given value of Λ (we have taken $\Lambda = 0.05$), the solutions of the continuum are denoted by red circles and the mixed solutions by green circles. The mixed solution branches are drawn using dotted lines. One observes multiplicity of solutions: at given Λ correspond several solutions with different B.

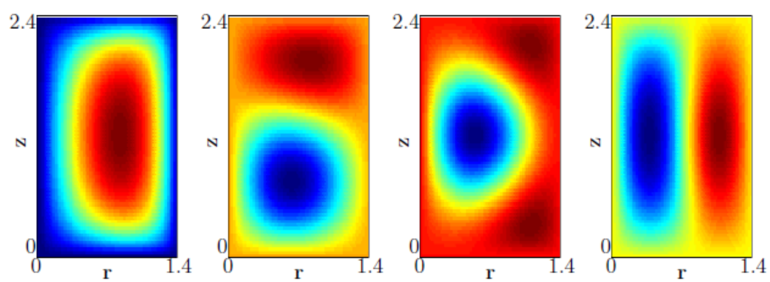


FIG. 2: Example of stream function ϕ of the four first solutions for $\Lambda = 0.05$. From left to right: $B = 0.3767$ (direct monopole), $B = 3.7874$ (vertical dipole), $B = 4.1633$ (reversed monopole) and $B = 5.15$. Increasing values from blue to red. By convention, we call direct (resp. reversed) monopole the one-cell solution with maximal (resp. minimal) inner stream function-see above. For simplicity, we show at each point only one solution, corresponding to a given sign of I. The solution corresponding to opposite sign of I can be found by a change $\phi \to -\phi$.

Entropy maximisation -> Beltrami flows

$$\omega \sim u_\theta$$

Conclusions

- La mécanique statistique permet de prédire la stabilité et l'émergence d'écoulements organisés.

- Régimes inertiels: faible forçage et dissipation

Domaines de validité:

- Turbulence 2D
- Turbulence géostrophique (multicouche)

Autres domaines:

- Turbulence MHD
- Systèmes gravitationnels (equation de Vlasov)
- Turbulence axisymétrique ?