

A la recherche d'une nouvelle taille de grains de
neige

Applications dans le domaine optique et micronde

Ghislain Picard – Marie Dumont

Physique nucléaire: Quelle distance parcourt, en moyenne, un neutron émis dans un réacteur ?



Ecologie: Quelle distance parcourt, en moyenne, une girafe dans un territoire donné ?



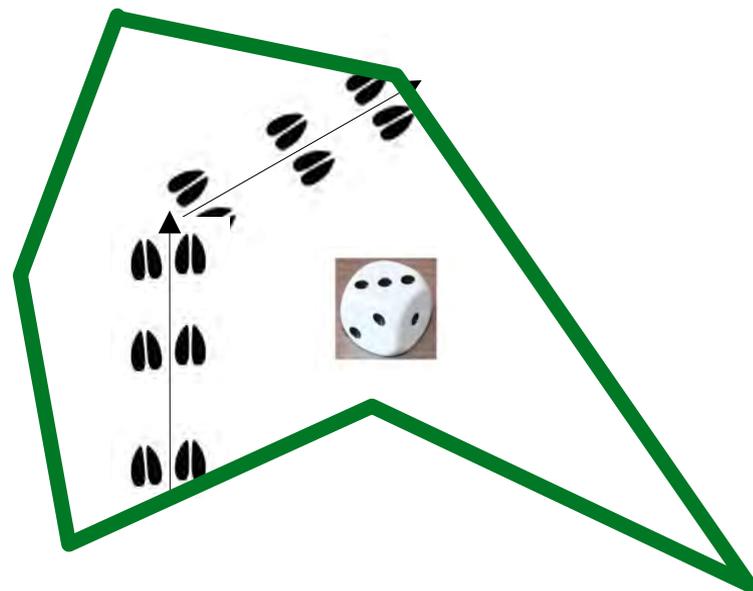
Ecologie: Quelle distance parcourt, en moyenne, une girafe d'une taille donnée dans un territoire donné?



Science participative: Quelle distance parcourt un chercheur qui déambule dans un espace ?

Expérience:

- tirer au hasard un nombre de pas
- entrer dans l'espace en un point au hasard, avec un angle au hasard
- avancer du nombre de pas
- tourner de façon totalement aléatoire
- avancer du nombre de pas
- continuer jusqu'à sortir de l'espace
- renseigner:
 - le nombre de pas
 - le nombre total de pas réalisés
 - mesurer la taille d'un pas en centimètre



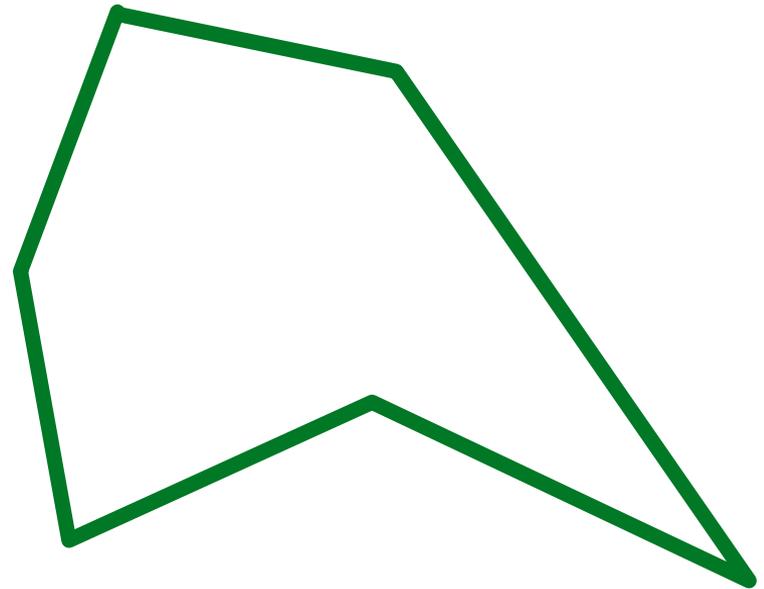
$$\langle l_n \rangle_{n=1\dots 6} = 3.19 \text{ m}$$

l = longueur parcourue

n = nombre de pas pour chaque segment du random walk

Notre hypothèse:

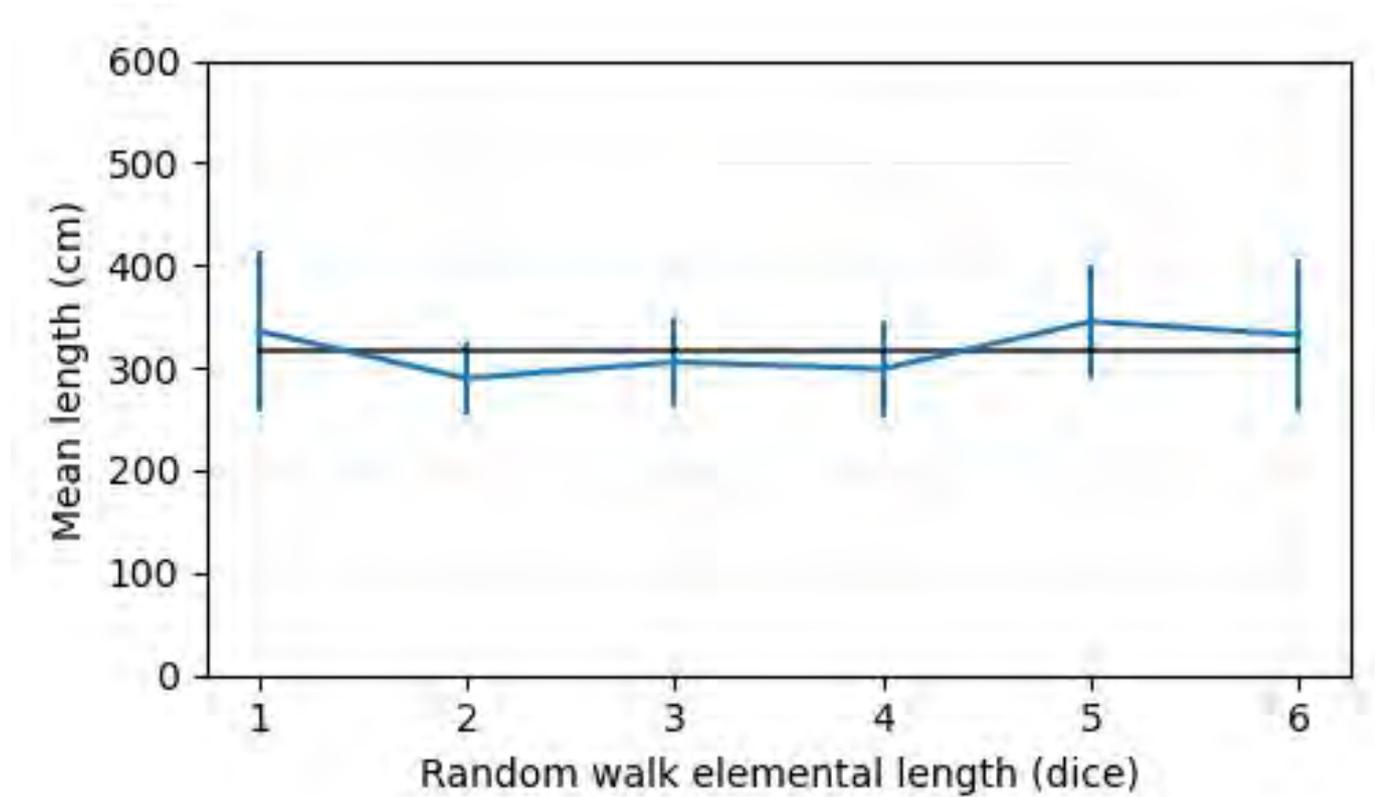
$\langle l_n \rangle$ est une excellente métrique pour
définir la "taille d'une zone"



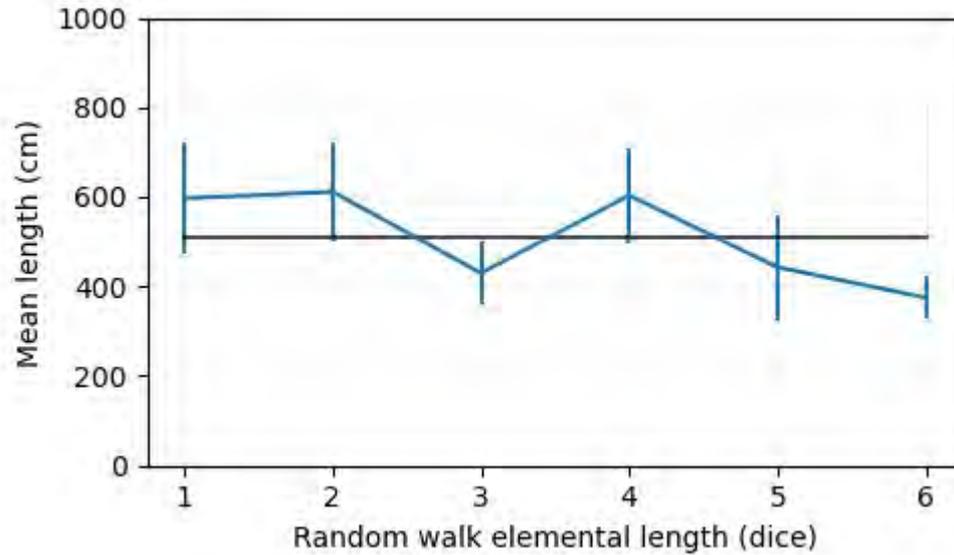
Question: comment se comparent les longueurs parcourues par les petites versus les grandes girafes ?

$$\langle l_n \rangle_{n=1} \text{ versus } \langle l_n \rangle_{n=6} ?$$

Histogramme des $\langle l_n \rangle$



Histogramme des $\langle l_n \rangle$



Expérience test du 9 mars 2025
(50 passages)

Ecologiquement:

On obtient la même valeur $\langle l_n \rangle$ pour les grandes girafes, les petites girafes ... et pour les fourmis.

Mathématiquement:

On obtient la même $\langle l_n \rangle$ pour une très large gamme de random walks

Physiquement:

On obtient la même longueur moyenne pour une large gamme de propagation diffusives / processus physiques aléatoires.



An invariance property of diffusive random walks

S. BLANCO and R. FOURNIER

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118 route de Narbonne, 31062 Toulouse Cedex 4, France*

(received 28 June 2002; accepted in final form 21 October 2002)

PACS. 05.60.Cd – Classical transport.

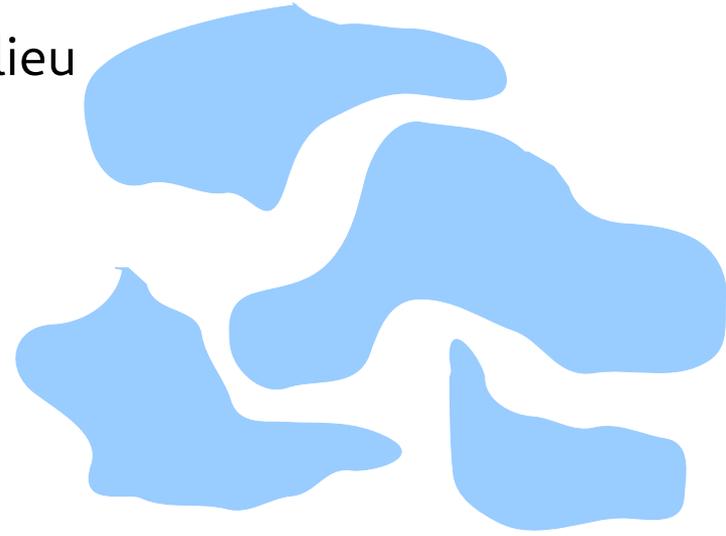
PACS. 05.40.-a – Fluctuation phenomena, random processes, noise, and Brownian motion.

Abstract. – Starting from a simple animal-biology example, a general, somewhat counter-intuitive property of diffusion random walks is presented. It is shown that for any (non-homogeneous) purely diffusing system, under any isotropic uniform incidence, the average length of trajectories through the system (the average length of the random walk trajectories from entry point to first exit point) is independent of the characteristics of the diffusion process and therefore depends only on the geometry of the system. This exact invariance property may be seen as a generalization to diffusion of the well-known mean-chord-length property (CASE K. M. and ZWEIFEL P. F., *Linear Transport Theory* (Addison-Wesley) 1967), leading to broad physics and biology applications.

$\langle l \rangle_{RW}$ est une excellente métrique
pour définir la "taille" d'une microstructure

Mean Path Length (MPL)

- pour une large gamme de random walks
- en 3D en 2D, pour des grains, pour un milieu poreux général

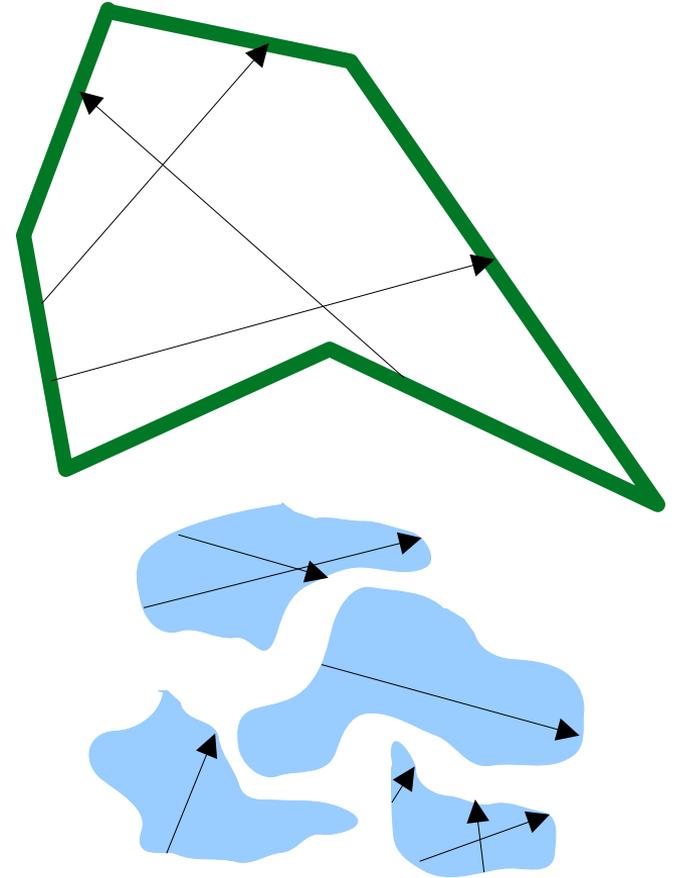


Doit-on abandonner la SSA ?

Si la longueur élémentaire du random walk est très grande, on obtient des lignes droites....

... qui sont dénommées usuellement "**cordes**".

Autrement dit: les cordes sont un cas particulier de random walk.



Si la longueur élémentaire du random walk est très grande, on obtient des lignes droites....

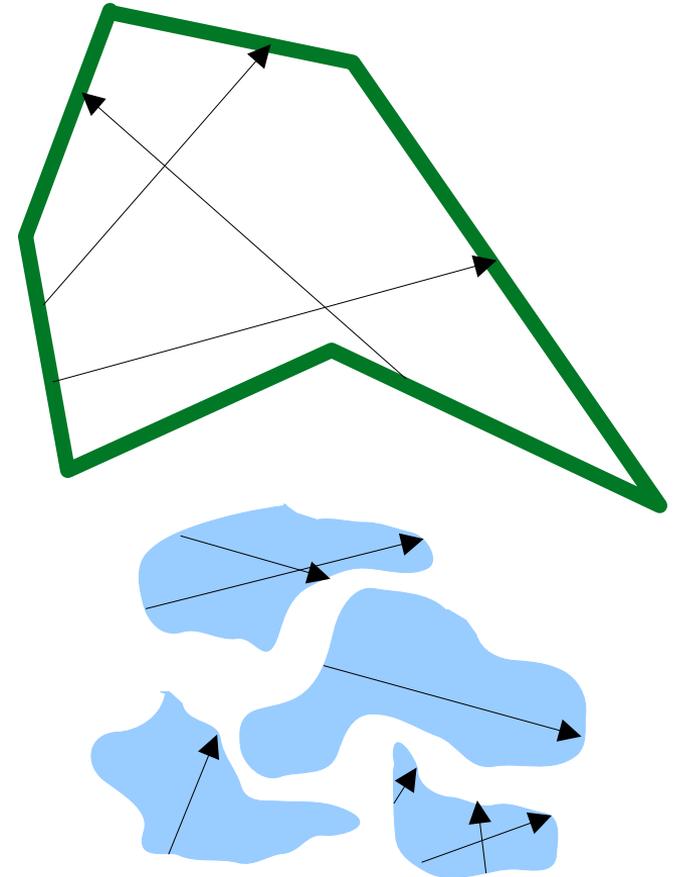
... qui sont dénommées usuellement "**cordes**".

Autrement dit: les cordes sont un cas particulier de random walk.

Le théorème de **Cauchy** (1789–1857), pour une forme convexe, donne:

$$\langle \text{longueur corde} \rangle = 4 V / S = 4 / (SSA \rho_{\text{ice}})$$

Autres noms: "theorem of the mean chord" or the "theorem of Dirac–Fucks" (1943, Manhattan project). Généralisation aux concaves Mazzolo et al. 2003



$$\text{MPL} = \langle l \rangle_{\text{RW}} = 4 / (\text{SSA } \rho_{\text{ice}})$$

pour une large gamme de random walks
pour les convexes et nombreux concaves

Short-Path Statistics and the Diffusion Approximation

Stéphane Blanco and Richard Fournier

We consider here a geometric system Ω of volume V with a boundary $\partial\Omega$ of surface S . Particles enter Ω uniformly and isotropically through $\partial\Omega$ and move across Ω along a trajectory of length l until their first exit through $\partial\Omega$. The random variable corresponding to the statistics of l is denoted L . It was shown in Ref. [1] that, for a three dimension diffusion random walk, the mean value of L is

$$\langle L \rangle = \frac{4V}{S} \quad (1)$$

independent of the random walk characteristics. $\langle L \rangle$ is therefore a purely geometric quantity: It is not modified when changing the fields of the local mean free path $[\lambda(\mathbf{x})$

at location \mathbf{x} for the average of the exponentially distributed paths between successive scattering events] and the single scattering phase function [the probability density function $p(\mathbf{u}_s, \mathbf{u}_i, \mathbf{x})$ of the scattering direction \mathbf{u}_s for an incident direction \mathbf{u}_i at \mathbf{x}]. Practical uses of this invariance property are reported in fields such as biology [2–4], colloid physics [5], radiative transfer [1,6,7], and neutronics [8]. It was also theoretically reexamined in the field of integral geometry as an extension of Cauchy’s formula [9,10]. Various further extensions were then proposed in Ref. [11]: General diffusion processes were considered,

Surprising variants of Cauchy's formula for mean chord length

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(Received 19 August 2019; published 19 November 2019)

We examine isotropic and anisotropic random walks which begin on the surface of linear (N), square ($N \times N$), or cubic ($N \times N \times N$) lattices and end upon encountering the surface again. The mean length of walks is equal to N and the distribution of lengths n generally scales as $n^{-1.5}$ for large n . Our results are interesting in the context of an old formula due to Cauchy that the mean length of a chord through a convex body of volume V and surface S is proportional to V/S . It has been realized in recent years that Cauchy's formula holds surprisingly even if chords are replaced by irregular insect paths or trajectories of colliding gas molecules. The random walk on a lattice offers a simple and transparent understanding of this result in comparison to other formulations based on Boltzmann's transport equation in continuum.

DOI: [10.1103/PhysRevE.100.050103](https://doi.org/10.1103/PhysRevE.100.050103)

Cauchy's theorem and generalization

Paul Reuss*

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Abstract. It has already been established that the mean length travelled by a neutral particle in a body containing a diffusing but not absorbing material is independent of its cross section, and consequently equal to the mean chord of the body. An elegant demonstration of this curious feature is presented and analysed thanks to Monte-Carlo simulations.

All the students in neutronics know the Cauchy's¹ theorem somehow related to the first collision probability theory. According to this theorem, the mean chord of a convex body is given by the very simple formula

$$\bar{X} = \frac{4V}{S}, \quad (1)$$

where V is the volume and S the surface of this body. The entry point A has to be chosen uniformly on the area of the surface and the inward direction isotropically. If B denotes the exit point, the chord X is the length of the segment AB .

Let us imagine the whole space completely empty but a stationary, uniform and isotropic flux Φ of monokinetic particles.^{3,4}

– The number of particles continuously being in V is

$$N = Vn = \frac{V\Phi}{v}. \quad (2)$$

– For an isotropic flux the number of particles crossing per unit of time and unit of surface element is

Φ

On comprend mieux pourquoi utiliser des cycloides au lieu des cordes permet aussi bien sinon mieux d'estimer la SSA

Stereological measurement of the specific surface area of seasonal snow types:
Comparison to other methods, and implications for mm-scale vertical profiling

M. Matzl, M. Schneebeli *

WSL Institute for Snow and Avalanche Research SLF, Flüelastr. 11, CH-7260 Davos-Dorf, Switzerland

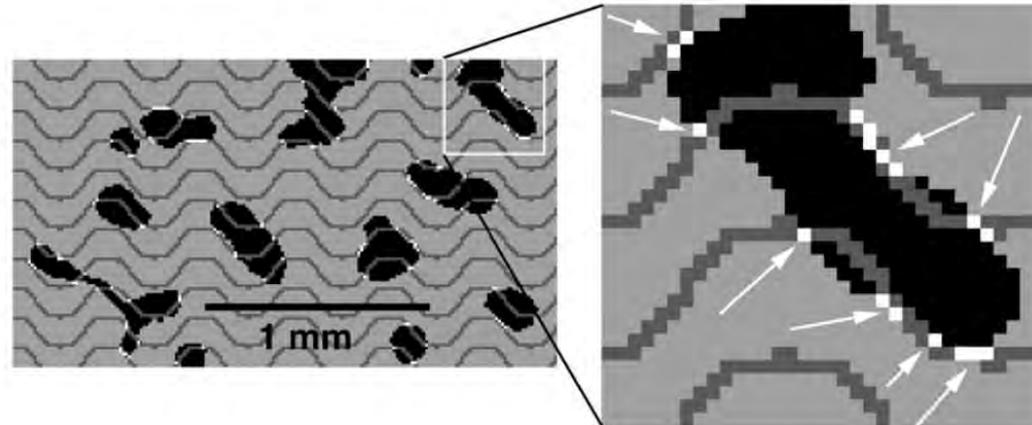


Fig. 2. Labeled intersections between the test system and the structure profiles: the ice grains are displayed in black. The light grey areas mark the pores and the dark grey lines the test lines. Each white arrow marks one intersection between the outlines of the ice grains and the test lines.

Conclusions:

On a une nouvelle définition très robuste, versatile et largement utilisée en science pour mesurer la taille de la microstructure de la neige.

On peut continuer à utiliser la SSA, qui est un “cas particulier”.

De nombreux processus physiques \Leftrightarrow propagation d'un random walk... Et la SSA est une grandeur naturelle pour décrire ces phénomènes.

Snow GS conversion table

SSA (kg m ⁻²)	MPL (μm)	r _{opt} (μm)
5	872	654
8	545	409
10	436	327
15	291	218
20	218	164
40	109	82
80	55	41

Perspectives:

Au delà de la moyenne $\langle l \rangle$ (MPL)... la distribution $p(l)$ et les moments statistiques $\langle l^n \rangle$ des longueurs (cordes, random walk plus général) contiennent des informations supplémentaires \rightarrow forme des grains.

“pour une large gamme de random walks”: quand la formule de Cauchy ne s'applique pas ?

Unraveling the optical shape of snow

Alvaro Robledano-Perez, Ghislain Picard, Marie Dumont, Frédéric Flin,
Laurent Arnaud et Quentin Libois

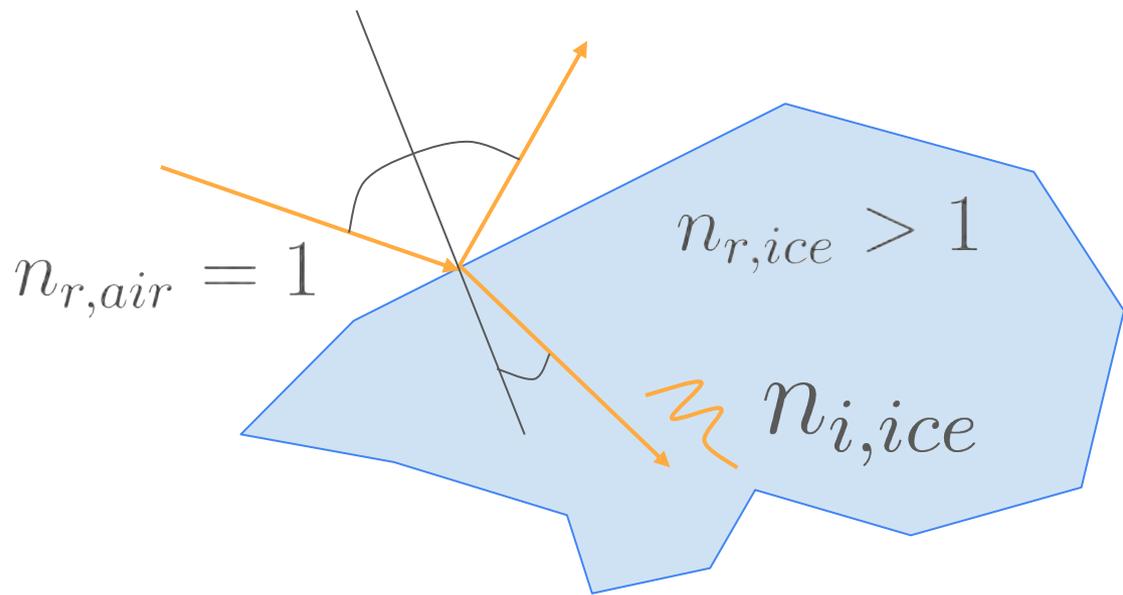
Nature communications, **14**, Article number: 3955 (2023)

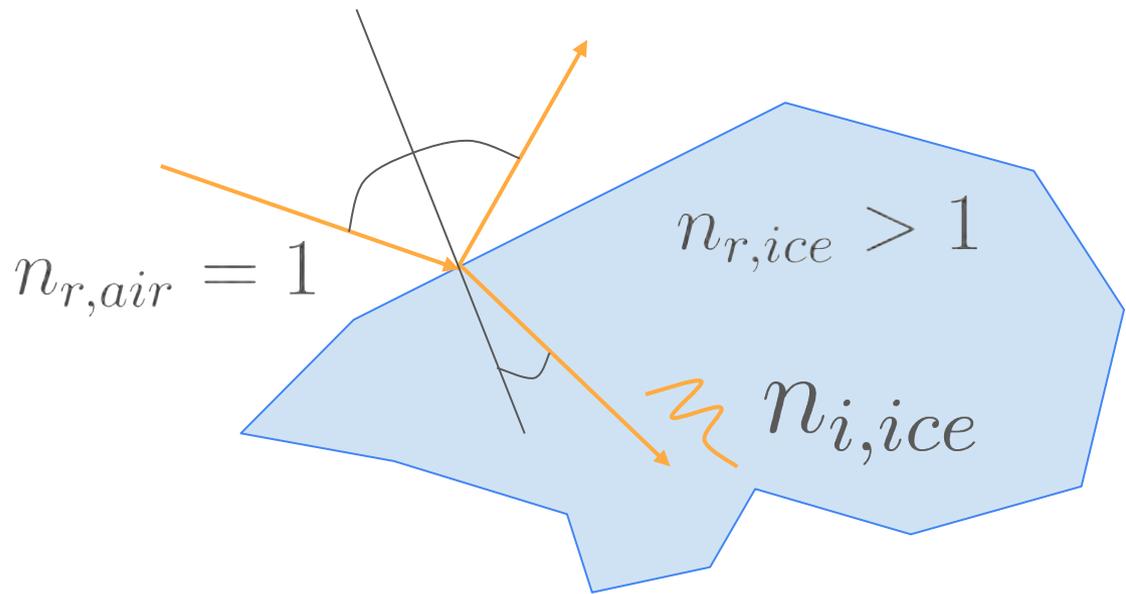


Picard (2009)

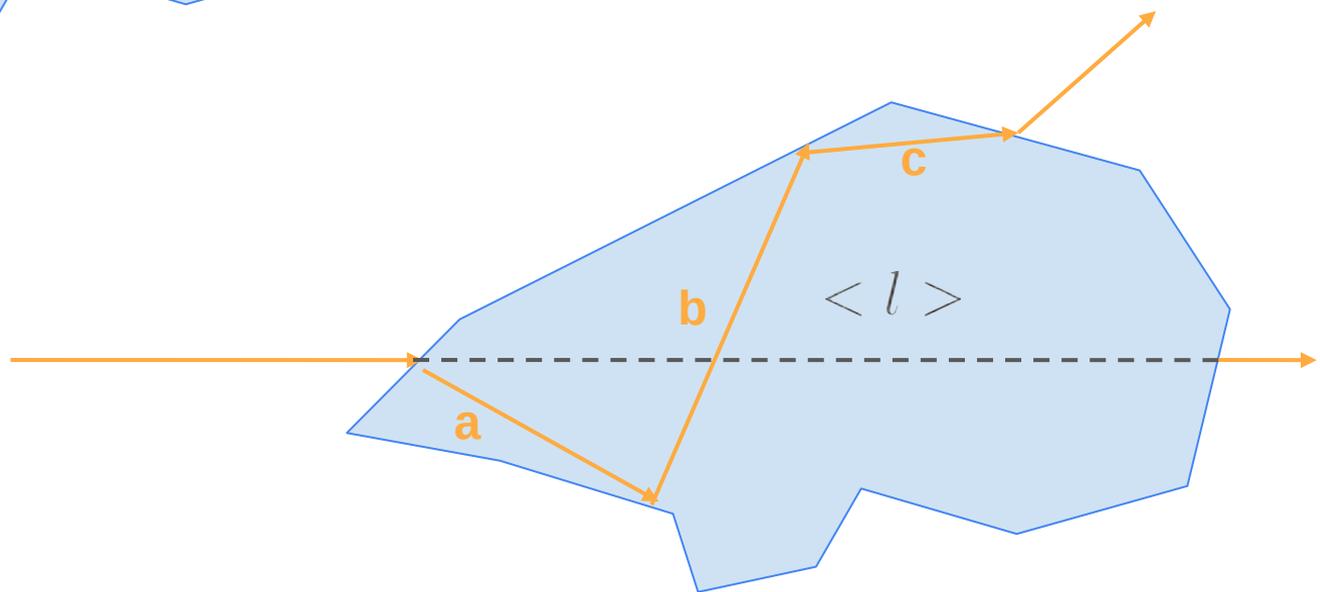


Col de Porte, IVORI





$$B \approx \frac{a + b + c}{\langle l \rangle}$$



$$l_{\text{photon}} = B \langle l \rangle$$

Bohren and Barkstrom, 1974 ; Kokhanovsky and Zege, 2004 ; Libois, 2014

$$l_{\text{photon}} = B \langle l \rangle$$

$$\alpha^2 \propto B_\lambda \frac{\langle l \rangle n_{i,ice,\lambda}}{\lambda}$$

Bohren and Barkstrom, 1974 ; Kokhanovsky and Zege, 2004 ; Libois, 2014

Mais que vaut B ?

500–1200). This gives a variability of less than 3%. We take the value $B \approx 1.27$ for estimations in this paper, which is similar to that used by Bohren³⁹ ($B \approx 1.26$). This means [Eq. (32)] that $\beta = B/n \approx 1.0$ (with errors smaller than 3%) for ice spheres in the visible, where $n \approx 1.31$. Also, we have on physical grounds $\beta \rightarrow 1$ as $n \rightarrow 1$ [or $C_{\text{abs}} \rightarrow \gamma V$ (Ref. 12)]. This means that β is highly robust against variations in the refractive index for the range of n relevant to snow optics problems.

The values of B for spheroids and hexagonal cylinders with various aspect ratios were tabulated in Refs. 12 and 40. In particular, it was found that $B = 1.2$ – 2.1 for spheroids and $B = 2.2$ – 2.5 for hexagonal cylinders if it was assumed that the ratio of their axes is in the range 0.5–2.0. The average of B for the interval 1.2–2.5 is 1.85, which is close to our estimation of $B \approx 1.84$ for fractal particles obtained from geometrical optics Monte Carlo calculations. The details of the code that we used to find B for fractals are given elsewhere.²⁸ The close correspondence of B for fractals and a mixture of particles that have various shapes is not surprising when one takes into account the fact that chaotic scattering and absorption by media with particles of diverse shapes should

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Mais que vaut B ?

1.6 ± 0.1

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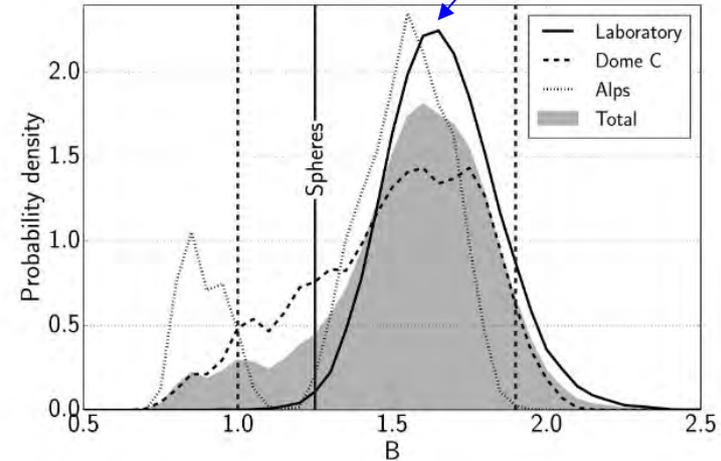
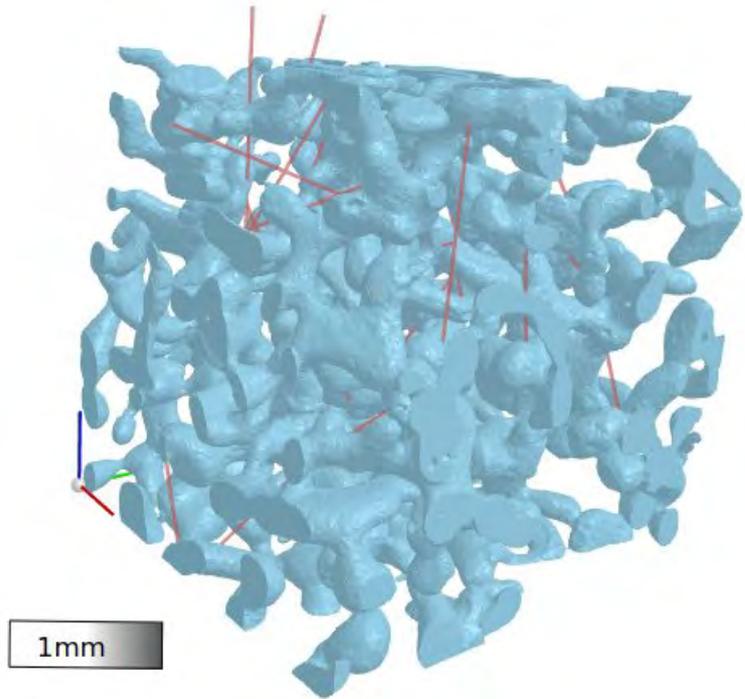
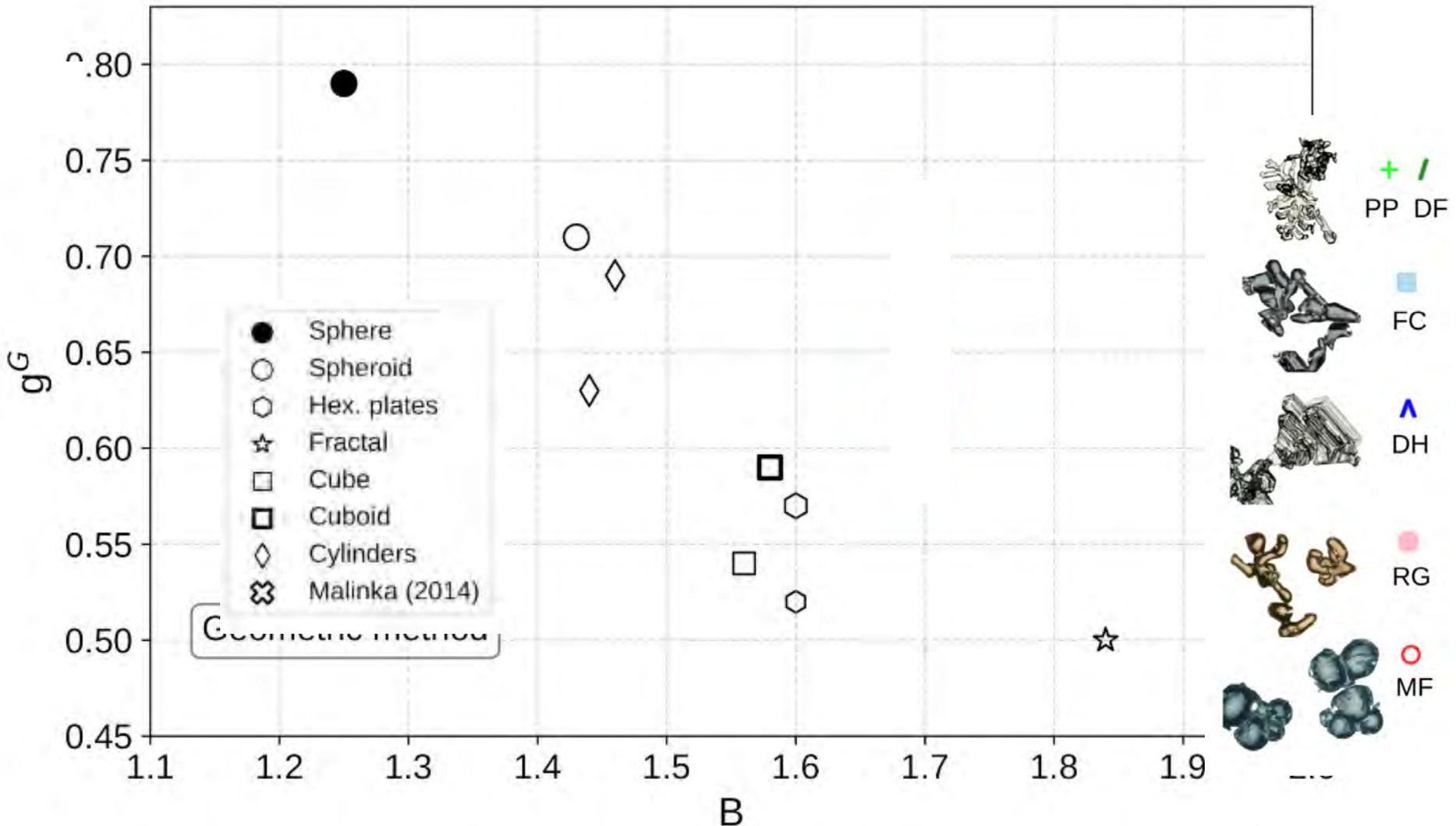


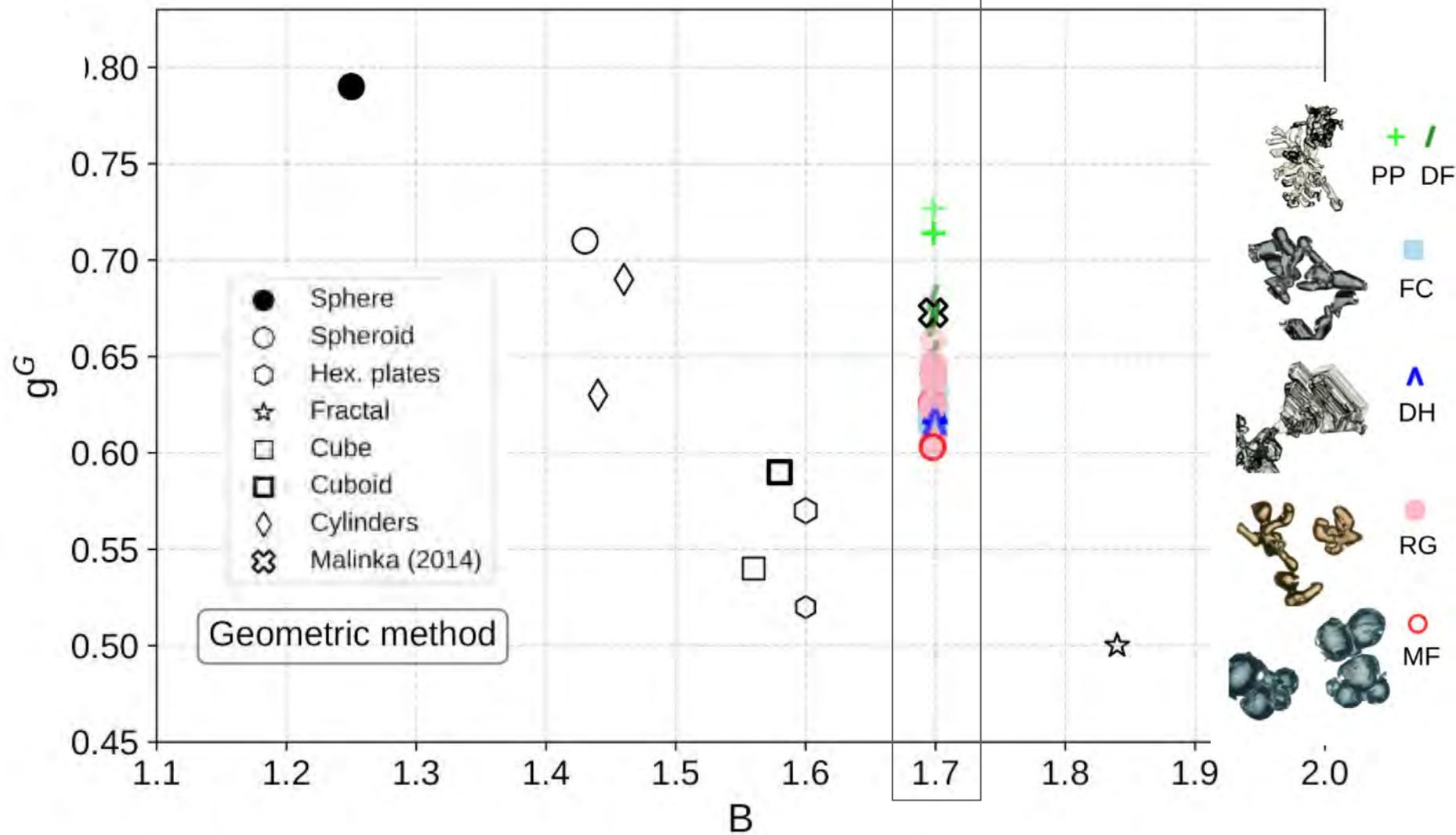
Fig. 8. Probability density function of B for all samples. The probability density functions for the laboratory, Dome C and Alps experiments are also shown individually. The vertical dashed bars show the 90% confidence interval.

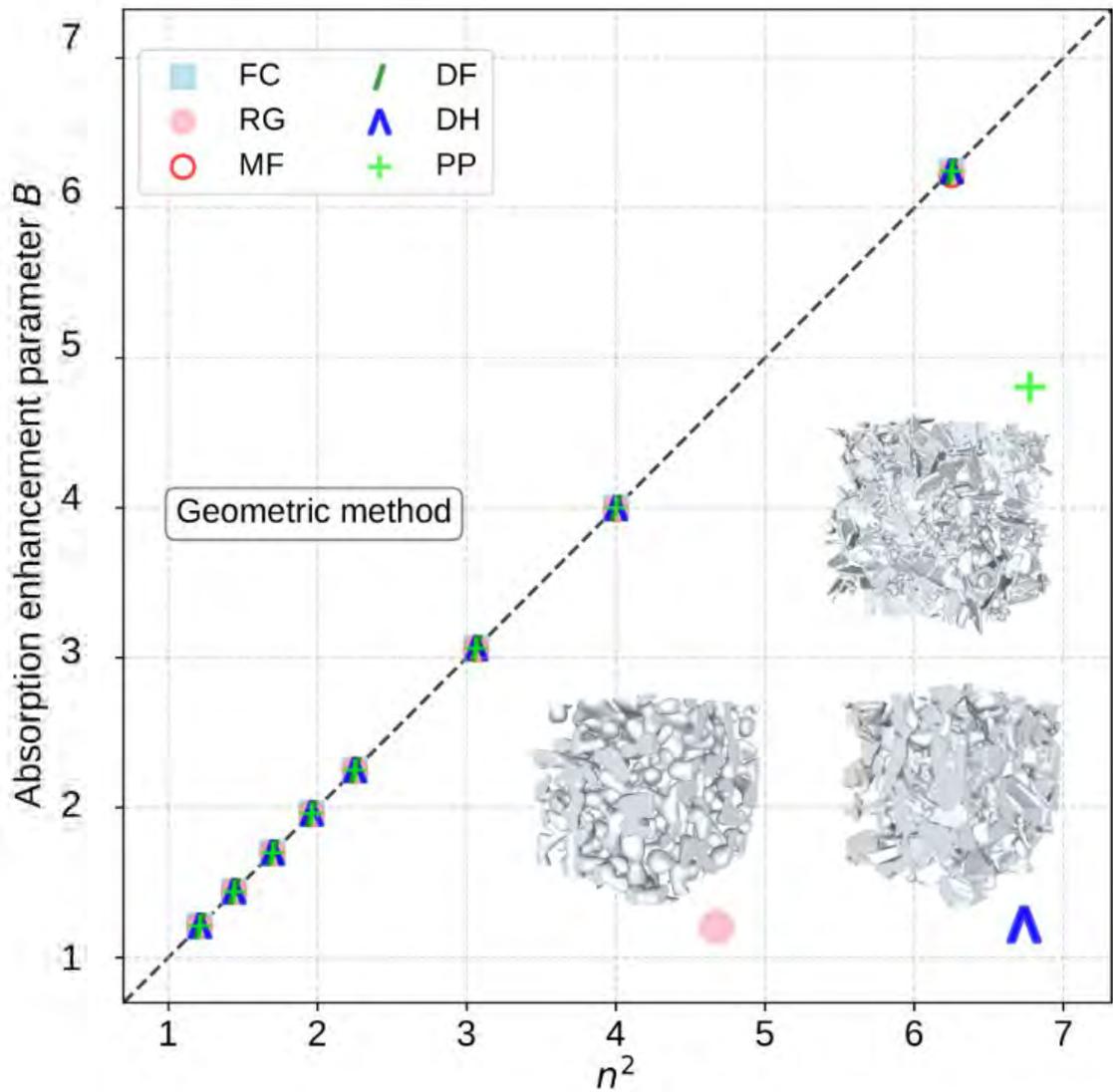


Robledano et al., 2022, 2023



Natural snow $B = 1.7$





Natural snow

$$B = n_r^2$$

500–1200). This gives a variability of less than 3%. We take the value $B \approx 1.27$ for estimations in this paper, which is similar to that used by Bohren³⁹ ($B \approx 1.26$). This means [Eq. (32)] that $\beta = B/n \approx 1.0$ (with errors smaller than 3%) for ice spheres in the visible, where $n \approx 1.31$. Also, we have on physical grounds $\beta \rightarrow 1$ as $n \rightarrow 1$ [or $C_{\text{abs}} \rightarrow \gamma V$ (Ref. 12)]. This means that β is highly robust against variations in the refractive index for the range of n relevant to snow optics problems.

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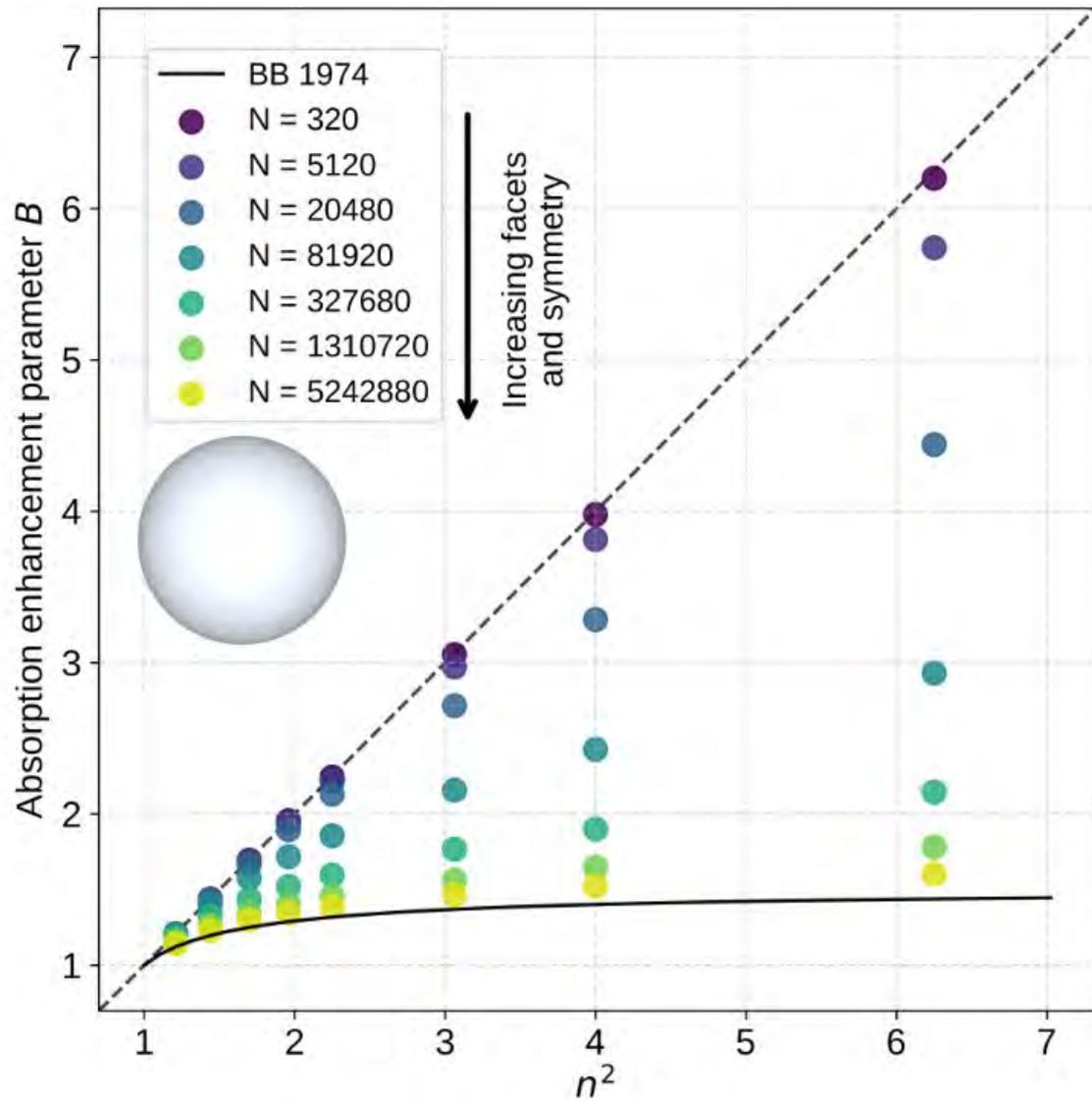
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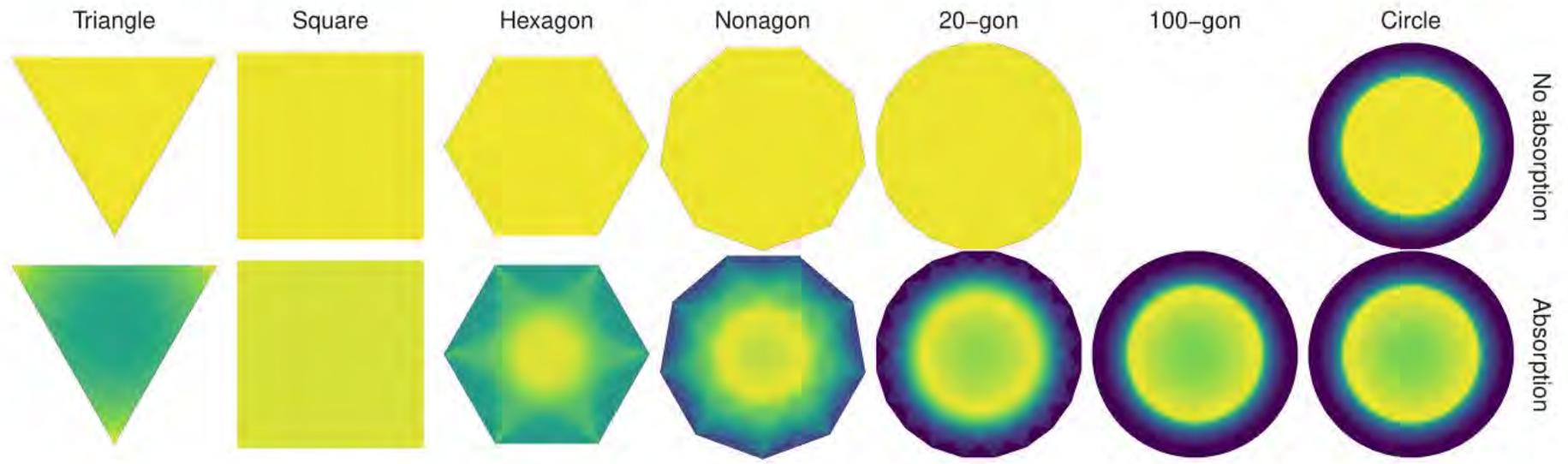
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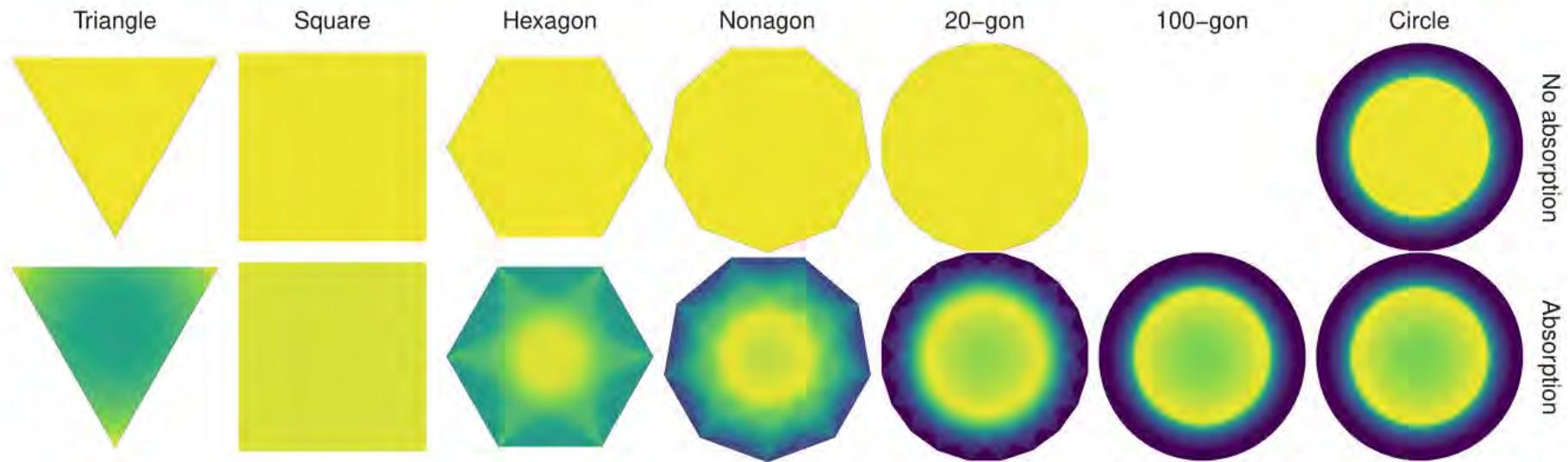
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$$B = n^2 * f(n, shape)$$



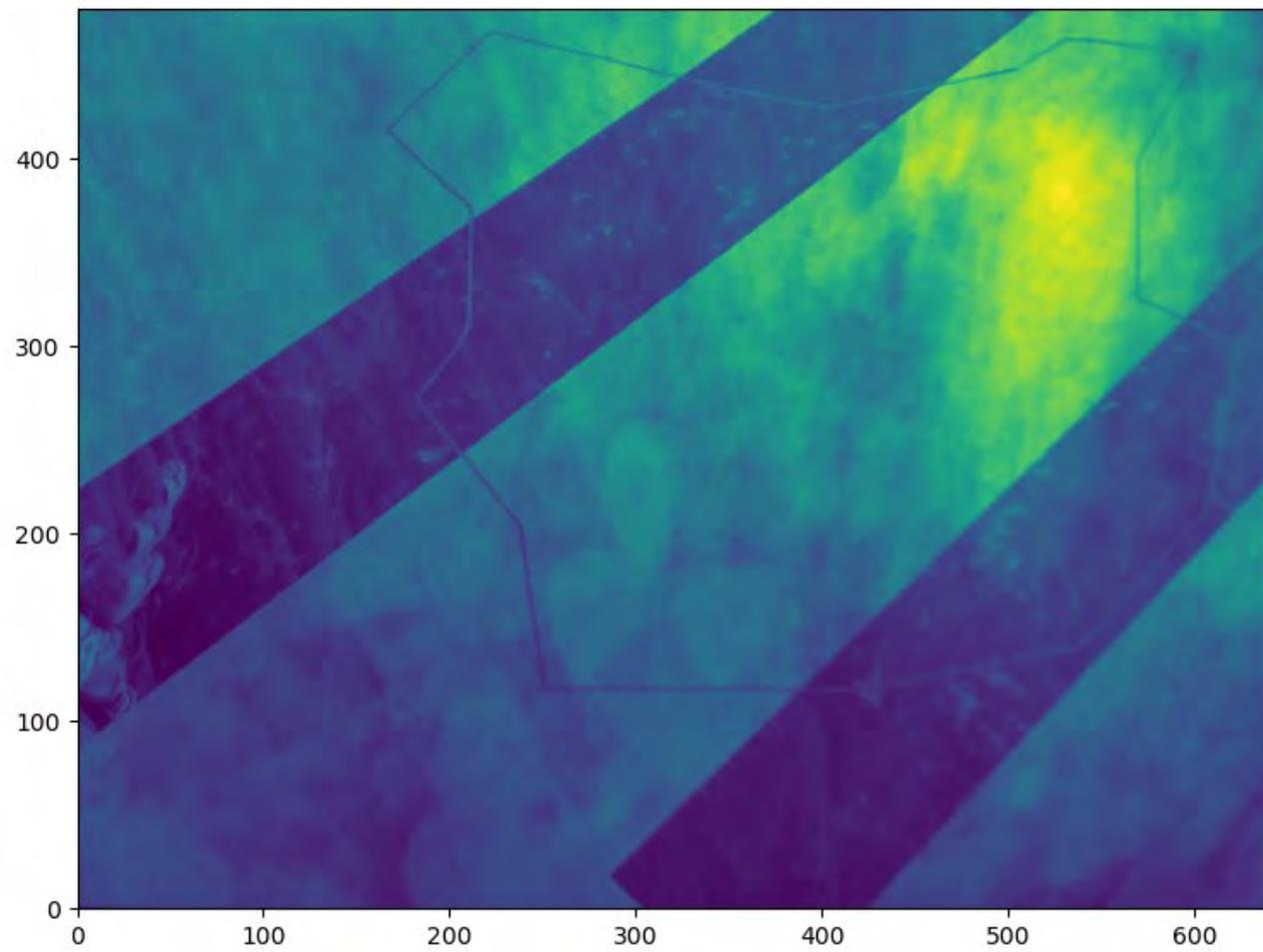


Majic et al., 2021



Majic et al., 2021

$$B = n^2 \left[1 - (\alpha\mu_1) \left(\frac{n^2}{T_{\text{out}}(n)} - 1 + \frac{\mu_2}{2\mu_1^2} \right) \right] \quad (\text{A5})$$



Science

OPTICS

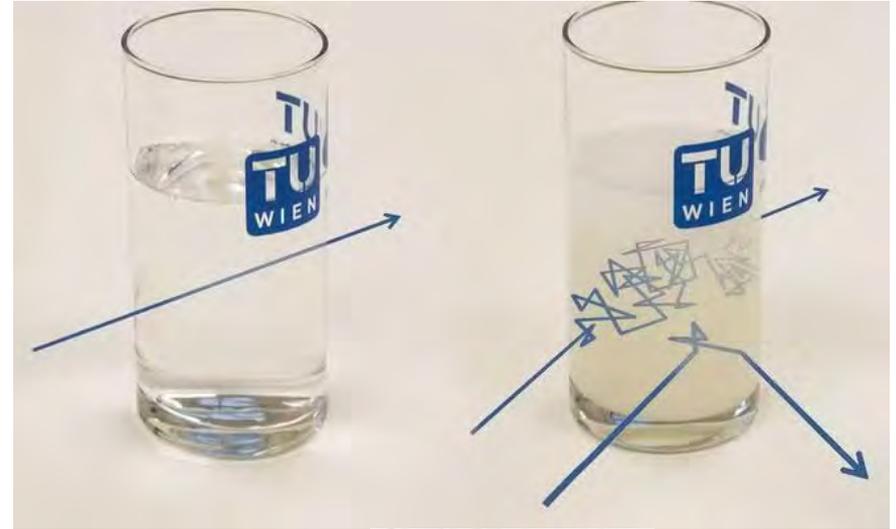
Observation of mean path length invariance in light-scattering media

Romolo Savo,¹ Romain Pierrat,² Ulysse Najar,¹ Rémi Carminati,²
Stefan Rotter,³ Sylvain Gigan^{1*}

The microstructure of a medium strongly influences how light propagates through it. The amount of disorder it contains determines whether the medium is transparent or opaque. Theory predicts that exciting such a medium homogeneously and isotropically makes some of its optical properties depend only on the medium's outer geometry. Here, we report an optical experiment demonstrating that the mean path length of light is invariant with respect to the microstructure of the medium it scatters through. Using colloidal solutions with varying concentration and particle size, the invariance of the mean path length is observed over nearly two orders of magnitude in scattering strength. Our results can be extended to a wide range of systems—however ordered, correlated, or disordered—and apply to all wave-scattering problems.

Savo et al., 2017

“Reprenons le verre de lait. Certes, comme la lumière est essentiellement réfléchiée, c'est qu'elle sort « vite » et a donc parcouru peu de distance dans le lait. Mais, à l'opposé de son entrée, une petite partie sort quand même, après cette fois de multiples diffusions, donc un long parcours. A l'inverse, dans un milieu plus transparent, il y aura beaucoup de chemins longs et peu de courts. Finalement, en moyenne, sur toutes les directions autour du contenant, on tombe sur une longueur ne dépendant que de la géométrie du contenant (plus précisément, le rapport de son volume à sa surface) et pas du détail de la composition du milieu diffusant !”



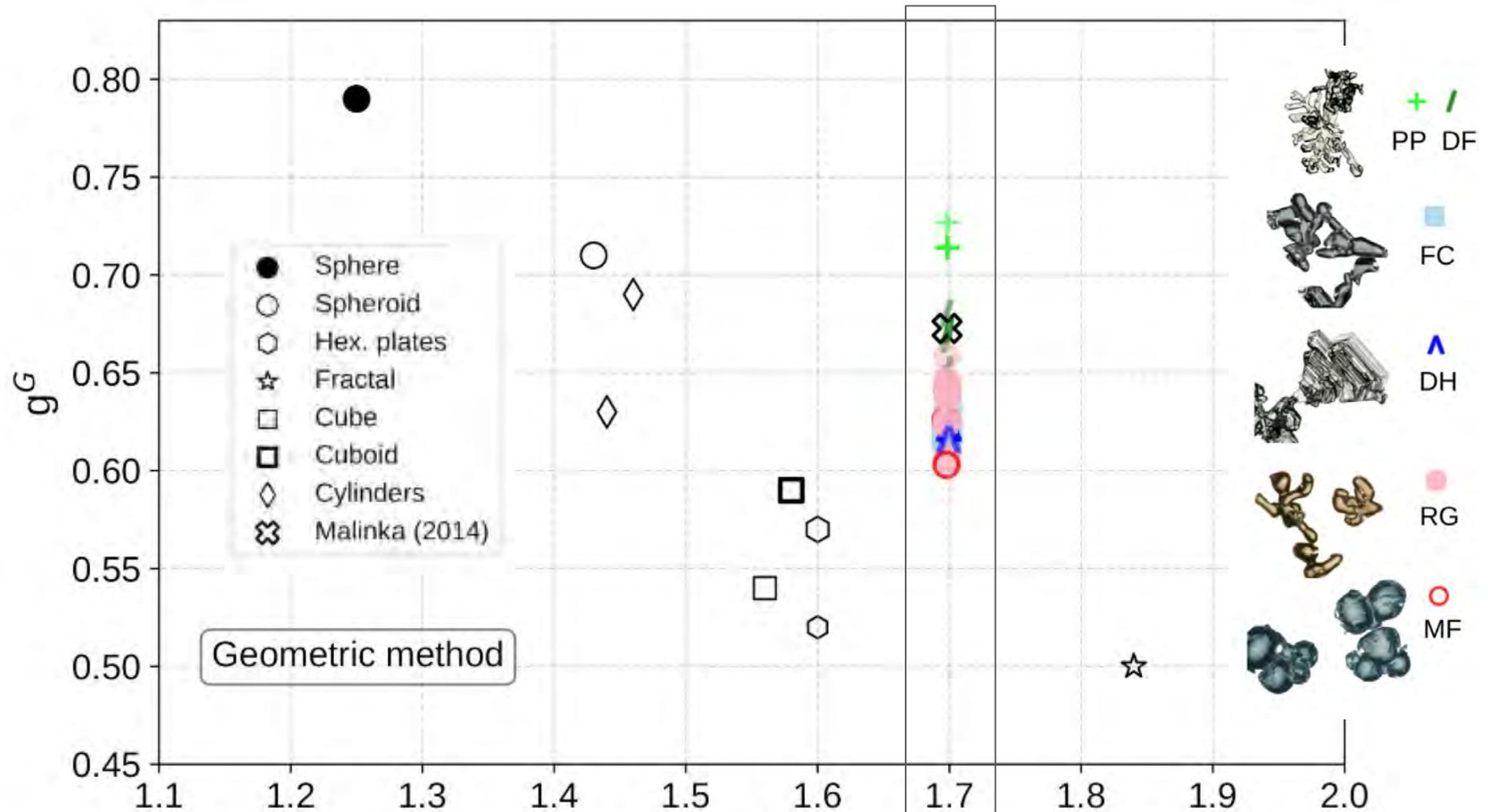
Le Monde, 2017

The snow microstructure is ergodic !

$$B = n^2$$

$$l_{\text{photon}} = n^2 \langle l \rangle$$

Natural snow $B = 1.7$



The microwave snow grain size:
a new concept to predict satellite observations
over snow-covered regions

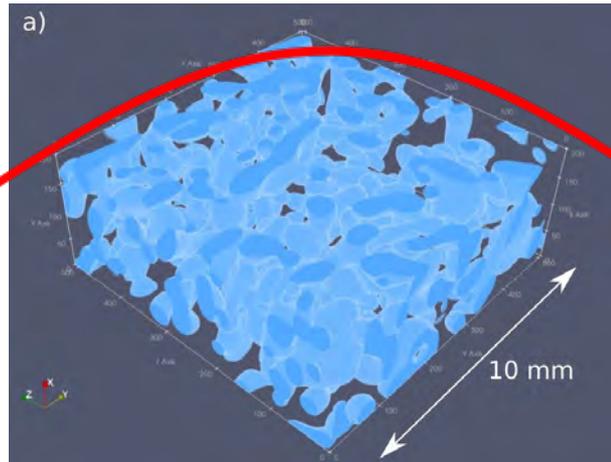
G. Picard, H. Löwe, F. Domine, L. Arnaud, F. Larue, V. Favier, E. Le Meur,
E. Lefebvre, J. Savarino, A. Royer

Comment les microondes interagissent avec la neige ?

Microonde = longueur d'onde de 3 - 300 mm

Les microondes ne voient pas chaque grain individuellement

MAIS sont très sensibles à la "taille des grains"

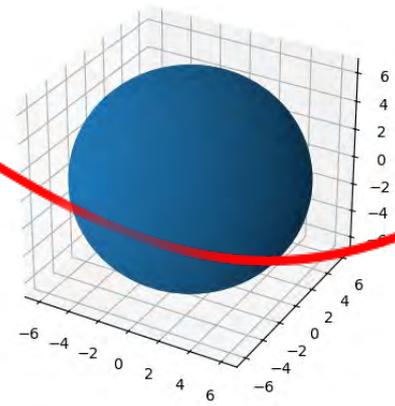
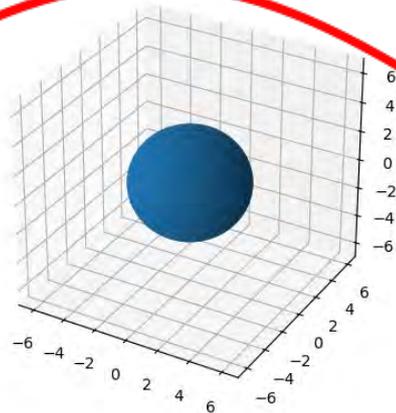
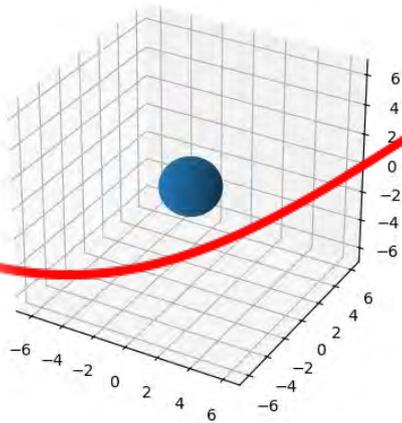


Des spheres indépendantes ... La diffusion de Rayleigh

le bleu du ciel et de la glace

volume des sphères

$$K_s = \frac{8\pi^3}{3} (\epsilon - 1)^2 \frac{N a^3}{\lambda^4}$$

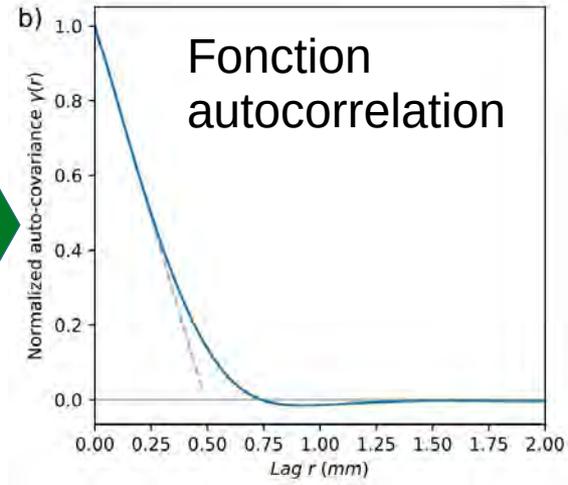
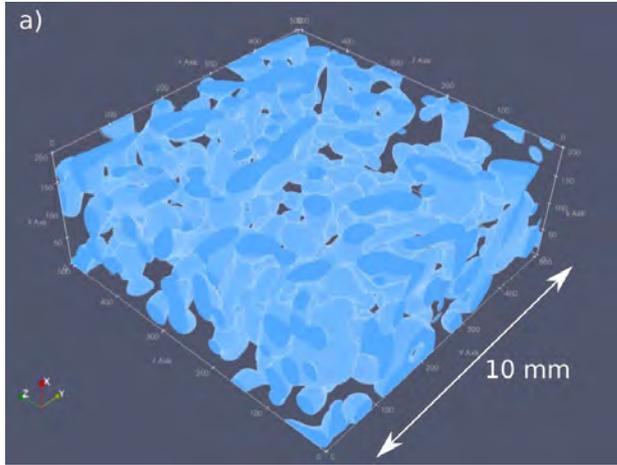


La neige est un milieu dense



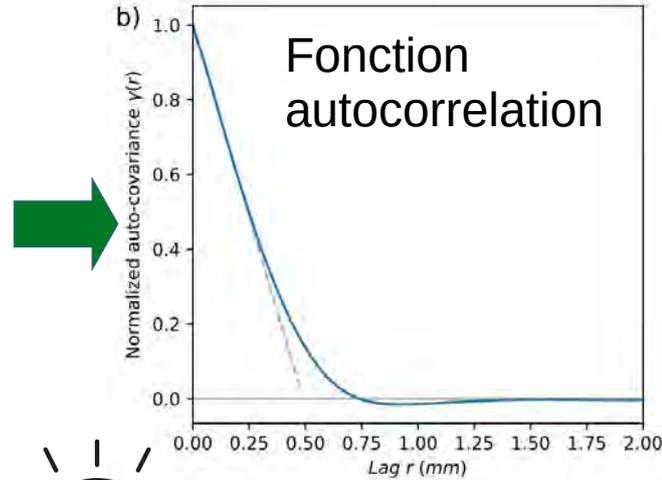
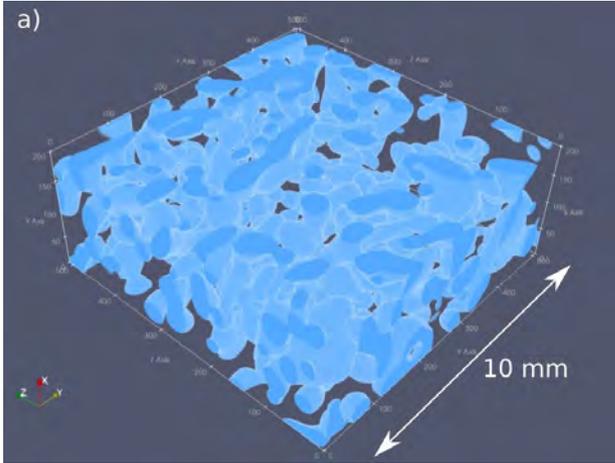
La neige est un milieu dense





Longueur de correlation,

Découverte de la “taille de grain microwave”



Longueur de corrélation,

Volume de corrélation

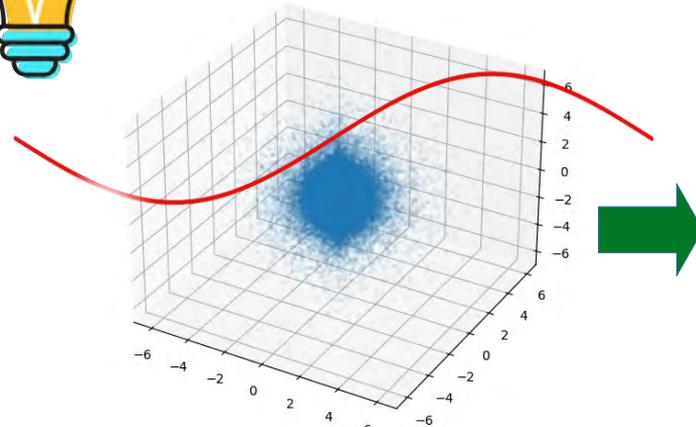


A lightbulb icon with rays emanating from it, symbolizing an idea or discovery.

Taille de grain microonde:

$$l_{\text{MW}} = \left(\frac{1}{2} \int_0^{\infty} \gamma(r) r^2 dr \right)^{1/3}$$

Ruland, W. (2010). Small-angle X-ray scattering of two-phase systems: Significance of polydispersity. *Journal of Applied Crystallography*, 43(5), 998–1004. <https://doi.org/10.1107/s0021889810031973>



Picard et al. 2022

Si on connaît l_{MW} , on détermine parfaitement la diffusion



Mais l_{MW} n'est pas mesurable !

parfait

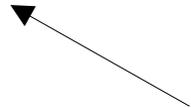


$$I_{MW} = K I_{Porod}$$



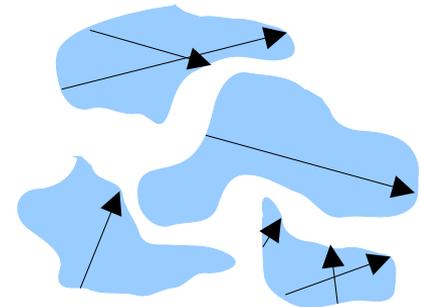
Qu'est-ce ?

mesurable



Ruland 2010 émet l'idée que la diffusion est contrôlée par **la polydispersité des cordes** dans le milieu

Ruland, W. (2010). Small-angle X-ray scattering of two-phase systems: Significance of polydispersity. *Journal of Applied Crystallography*, 43(5), 998–1004. <https://doi.org/10.1107/s0021889810031973>



$\langle l^4 \rangle$ Moyenne des longueurs de corde ⁴

$$K = \left(\frac{\mu_4}{24\mu_1^4} - \frac{\mu_2\mu_3}{6\mu_1^5}\phi + \frac{\mu_2^3}{8\mu_1^6}\phi^2 \right)^{1/3} (1 - \phi)^{-2/3}$$

$\langle l \rangle$ Longueur moyenne des cordes

La polydispersité microonde K est fonction des 4 moments de la distribution de longueur de corde et de la densité (ϕ)

Encore les formules de **Cauchy** :

$$\langle l \rangle = \frac{4V}{S}$$

$$\langle l^4 \rangle = \frac{12V^2}{\pi S}$$

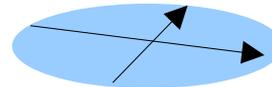
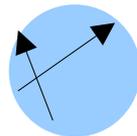


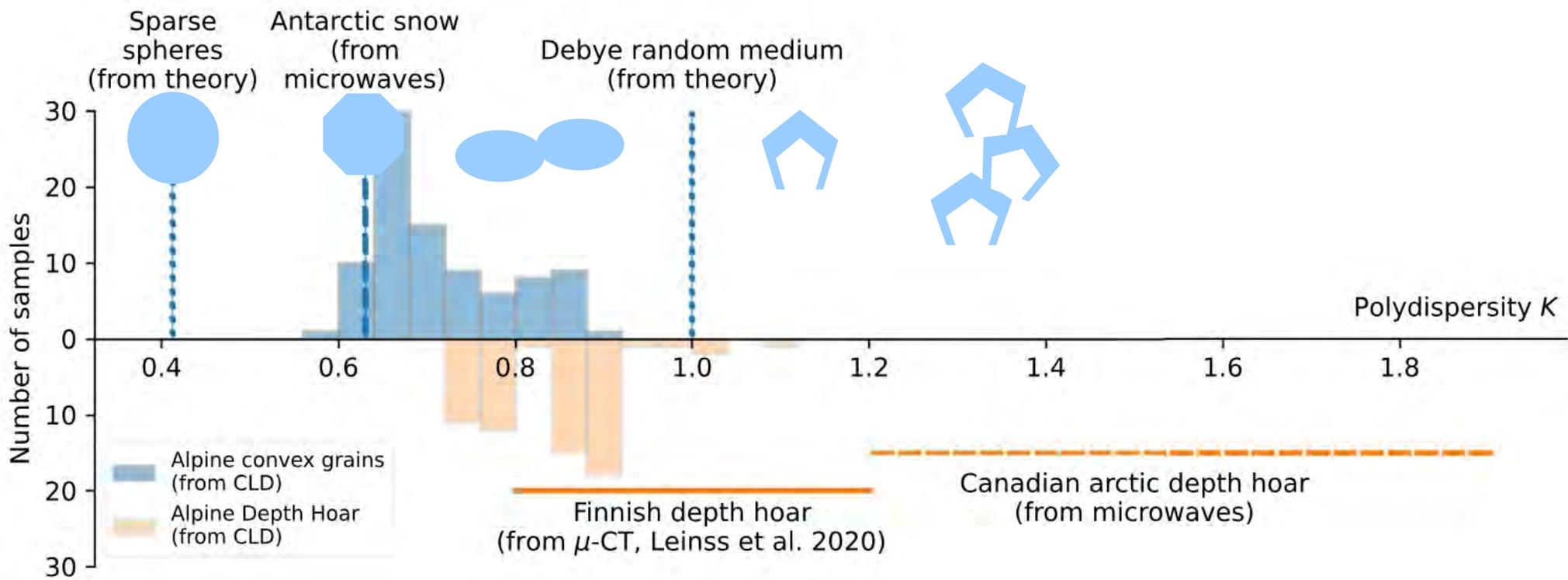
$$K_1 \propto \frac{S}{V^{2/3}}$$



= isoperimetric shape factor (à une puissance prêt)

Indicateur de sphéricité





Conclusions:

La taille de grain microonde est bien définie et idéale pour la diffusion MW.

La longueur de Porod est mesurable (SSA et densité).

La polydispersité microonde permet de faire le lien.

- on a une formule générale
- on a un début d'interprétation
- on a quelques mesures

$$\text{MW grain size} = \text{MW polydispersity} \times \text{porosity} \times \text{MPL}$$

Perspectives:

Calculer K pour les tomographies existantes

et les mesures conjointes MW et tomo

