

Modeling the Transport of Suspended Particles Using LES

Accounting for Finite-Size Effects in a Continuum Model

ATELIER TURBULENCE DE L'OSUG | 2ÈME JOURNÉE

DECEMBER 5, 2024

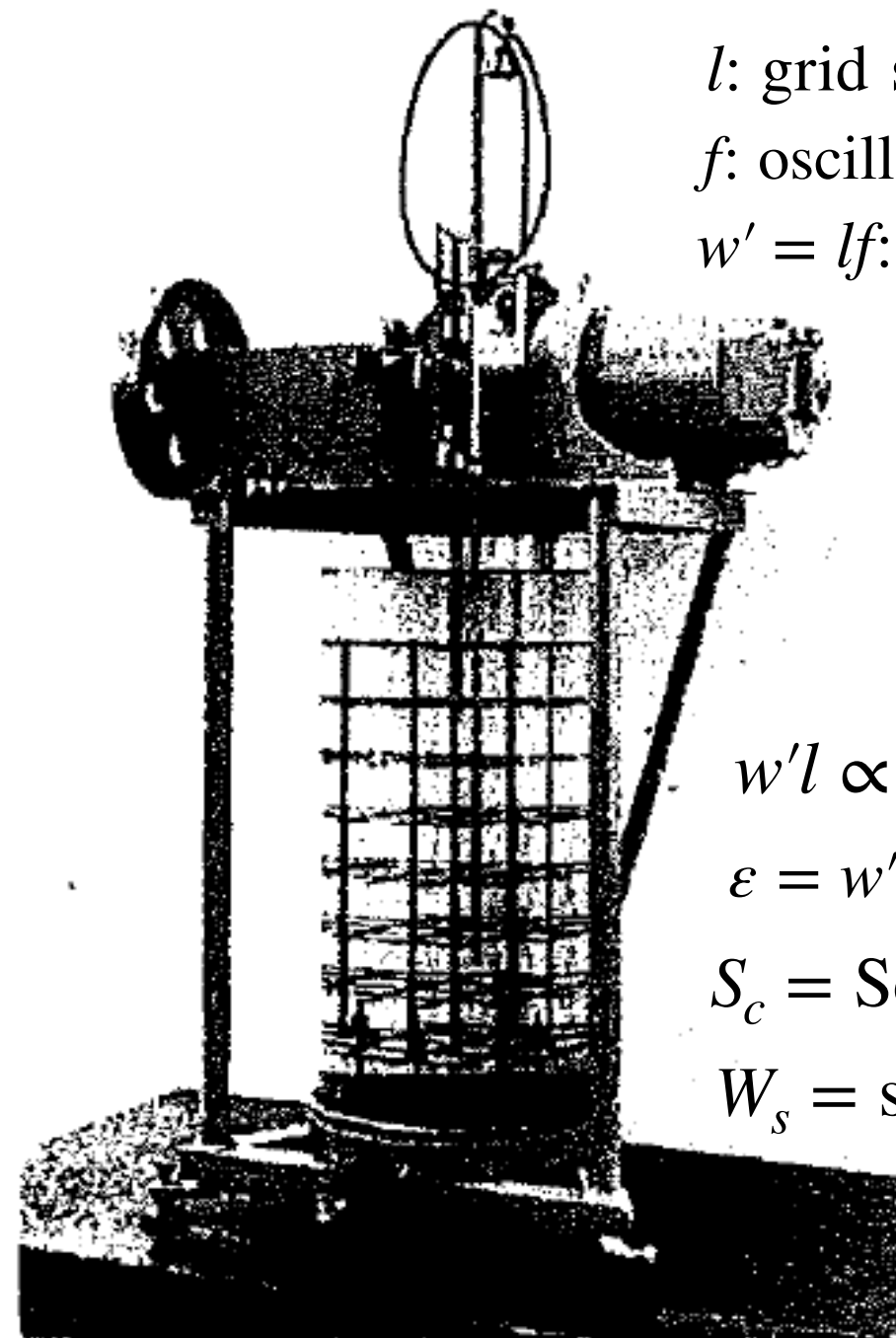
Julien CHAUCHAT

*Associate Professor
University Grenoble Alpes
LEGI, Grenoble, France
Industrial chair of excellence OXALIA*



*Senior Researcher @ INRAE
University Grenoble Alpes
IGE, Grenoble, France*

HUNTER ROUSE EXPERIMENTS (ICTAM 1938)



l : grid spacing (m)
 f : oscillation freq. (Hz)
 $w' = lf$: vel. fluc. (m/s)

$$w'l \propto l^2 f = \text{cst}$$

$$\varepsilon = w'l/S_c$$

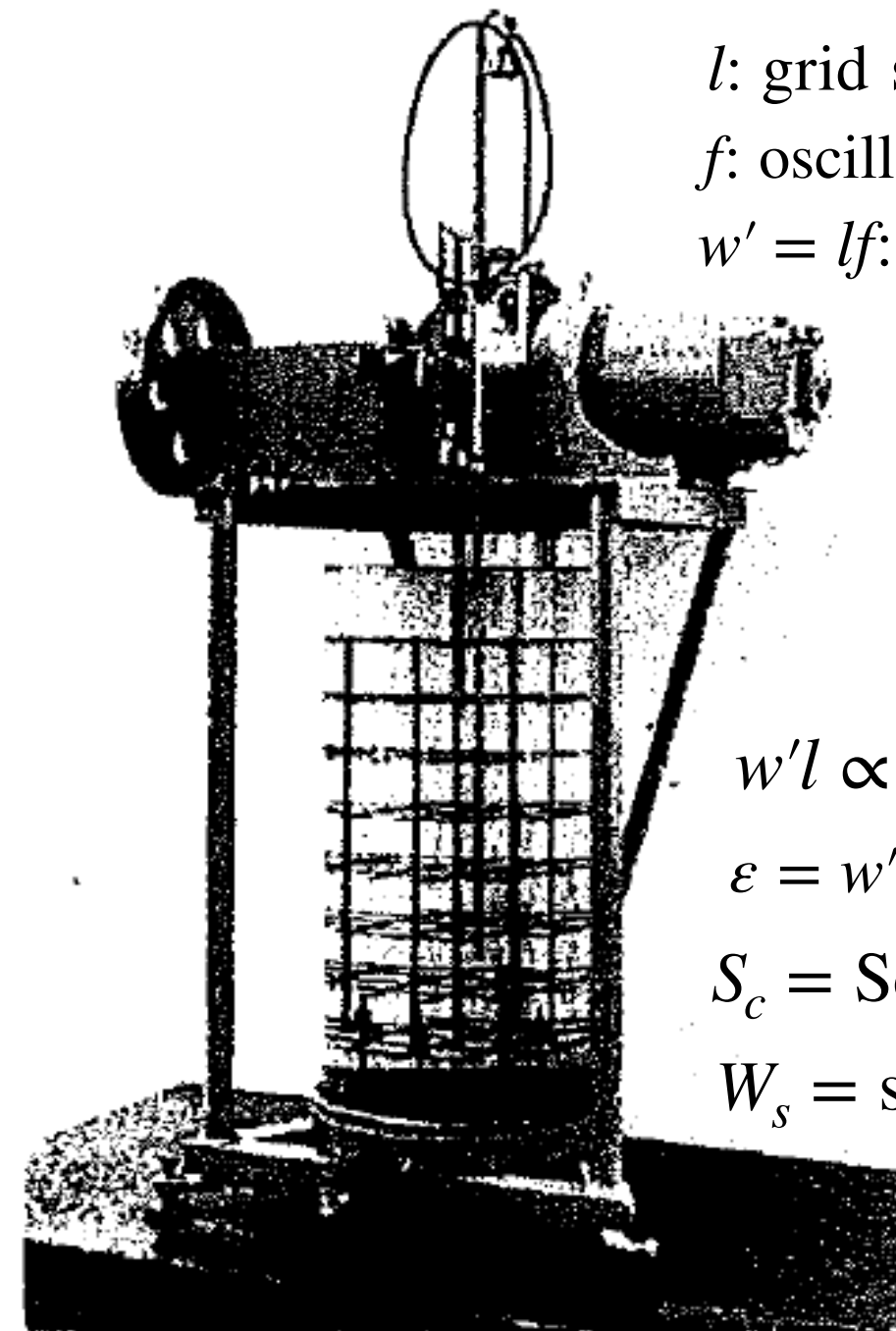
S_c = Schmidt number

W_s = settling velocity

Fig.1 General view of experimental apparatus.

Nominal diameter d	Size range	Sieve mesh	Settling velocity	Reynolds number
1/4 mm	.351 -- .246 mm	42 -- 30	3.69 cm/s	11.0
1/8 mm	.175 -- .124 mm	80 -- 115	1.75 cm/s	2.6
1/16 mm	.088 -- .061 mm	170 -- 250	0.675 cm/s	0.5
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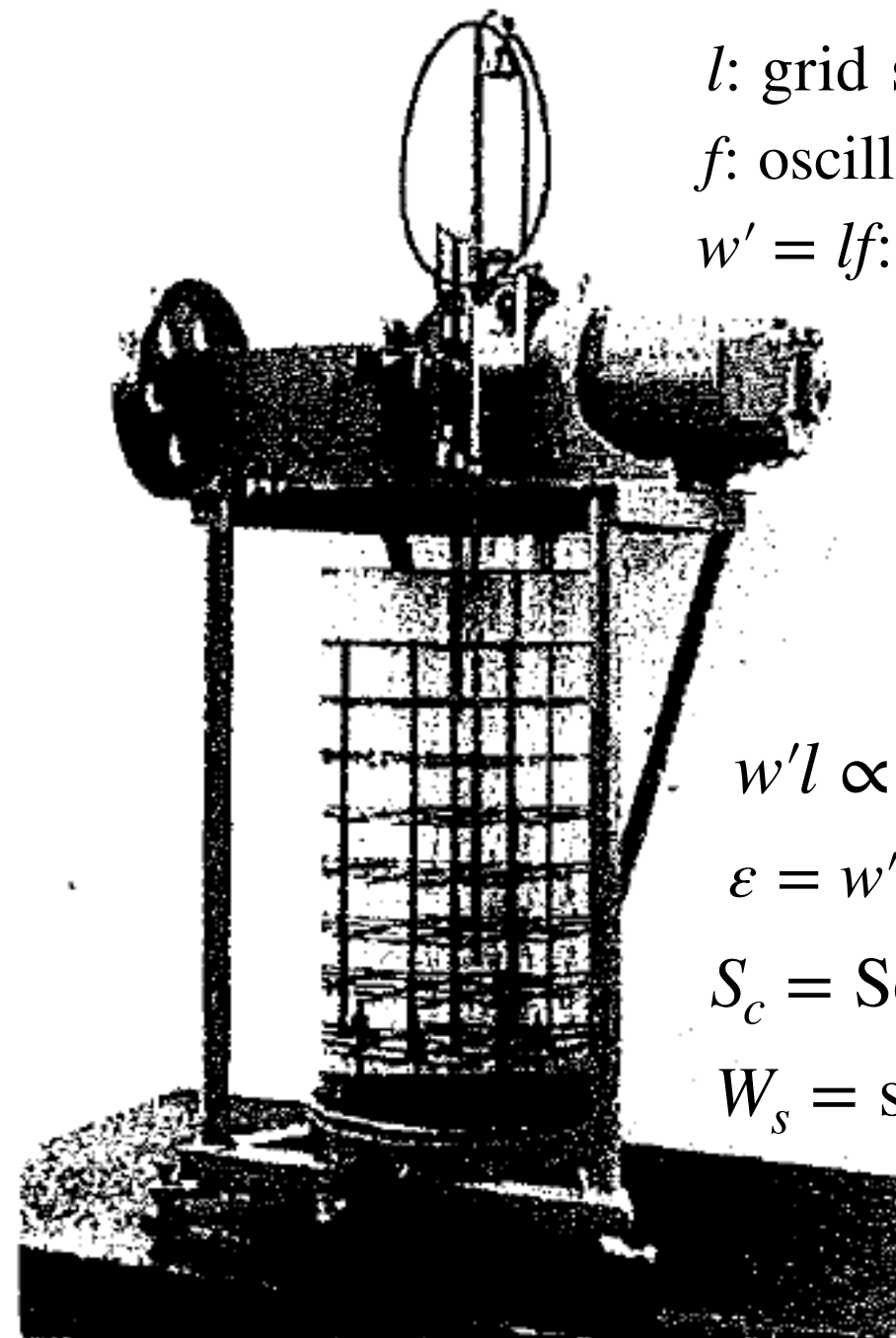
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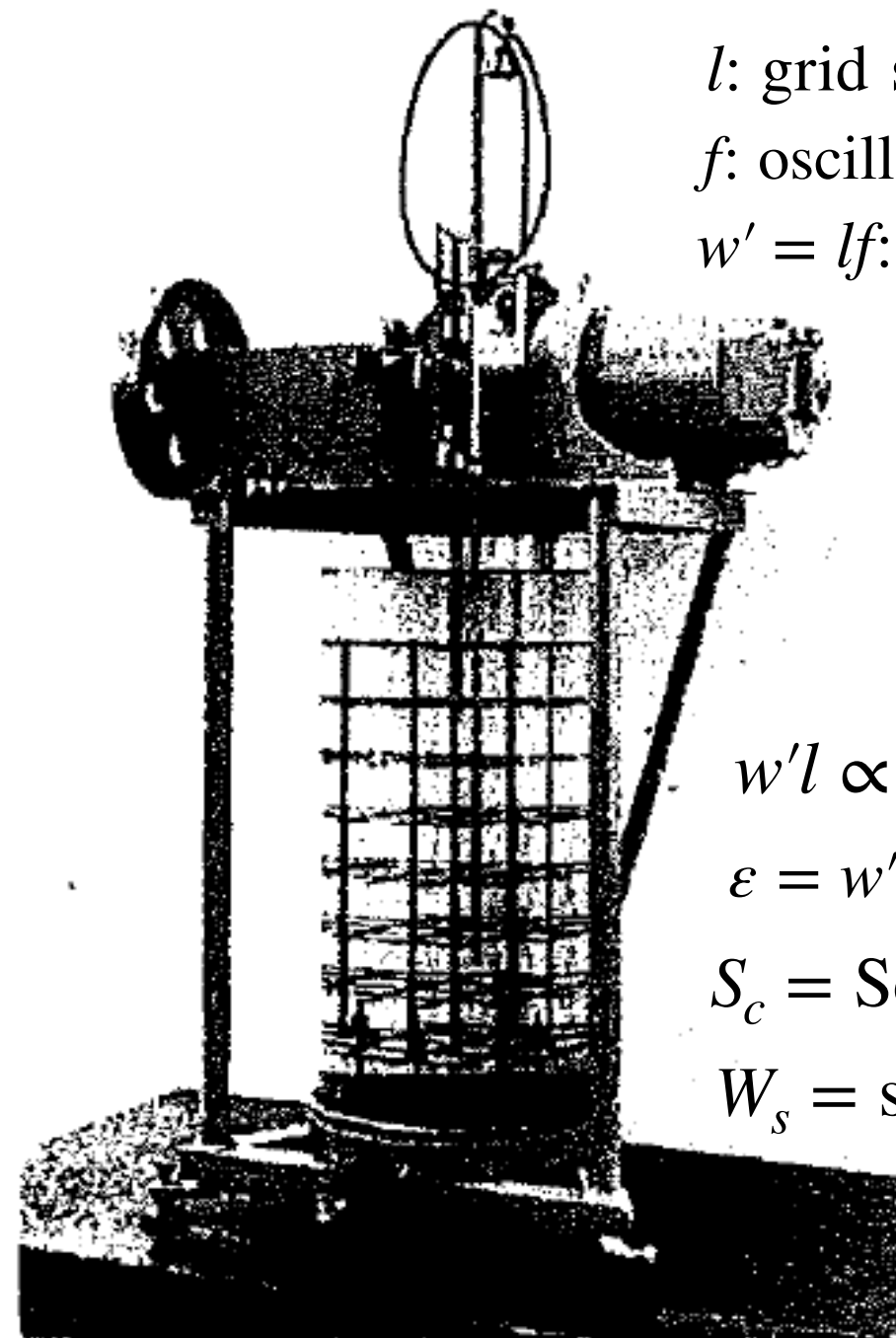
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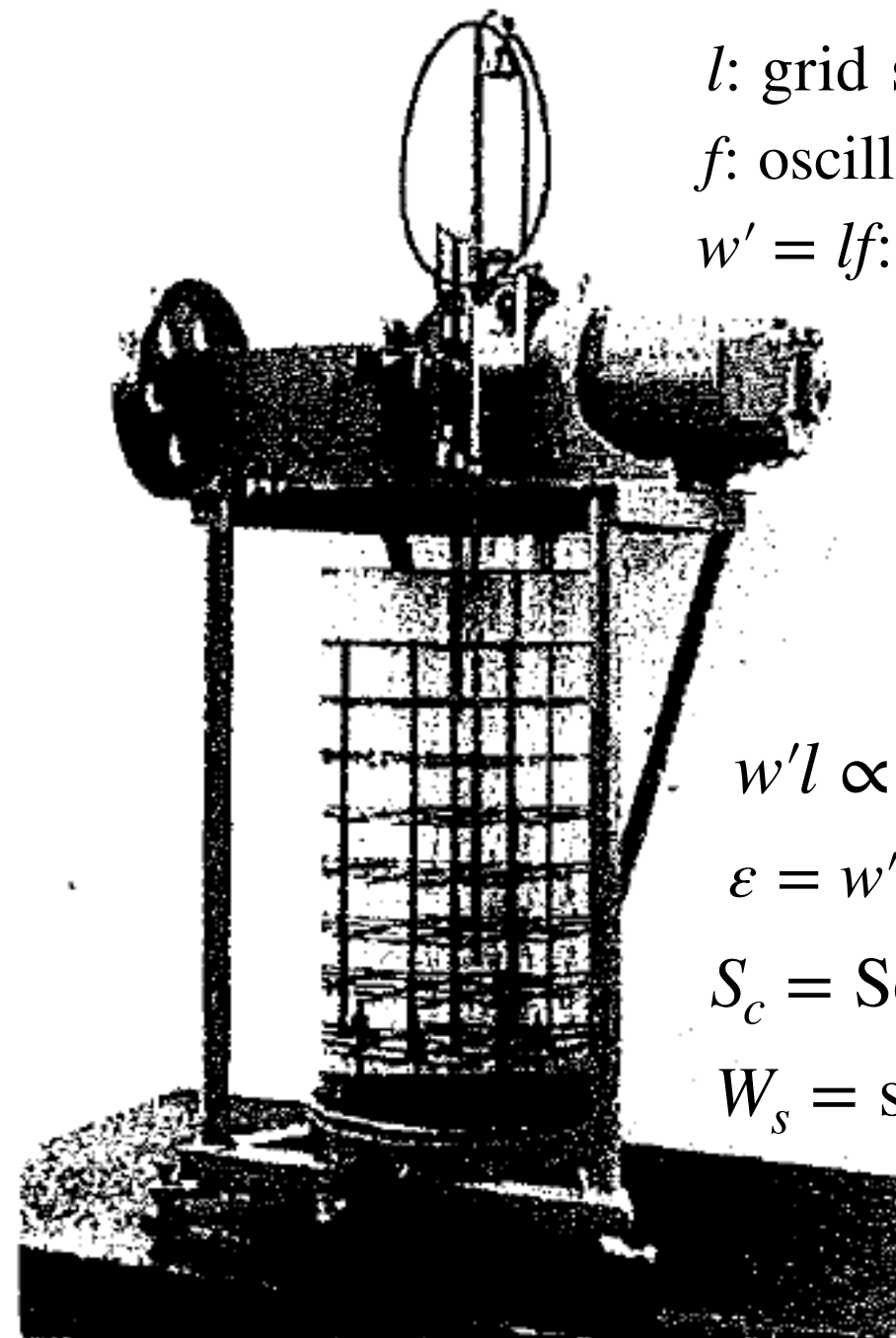
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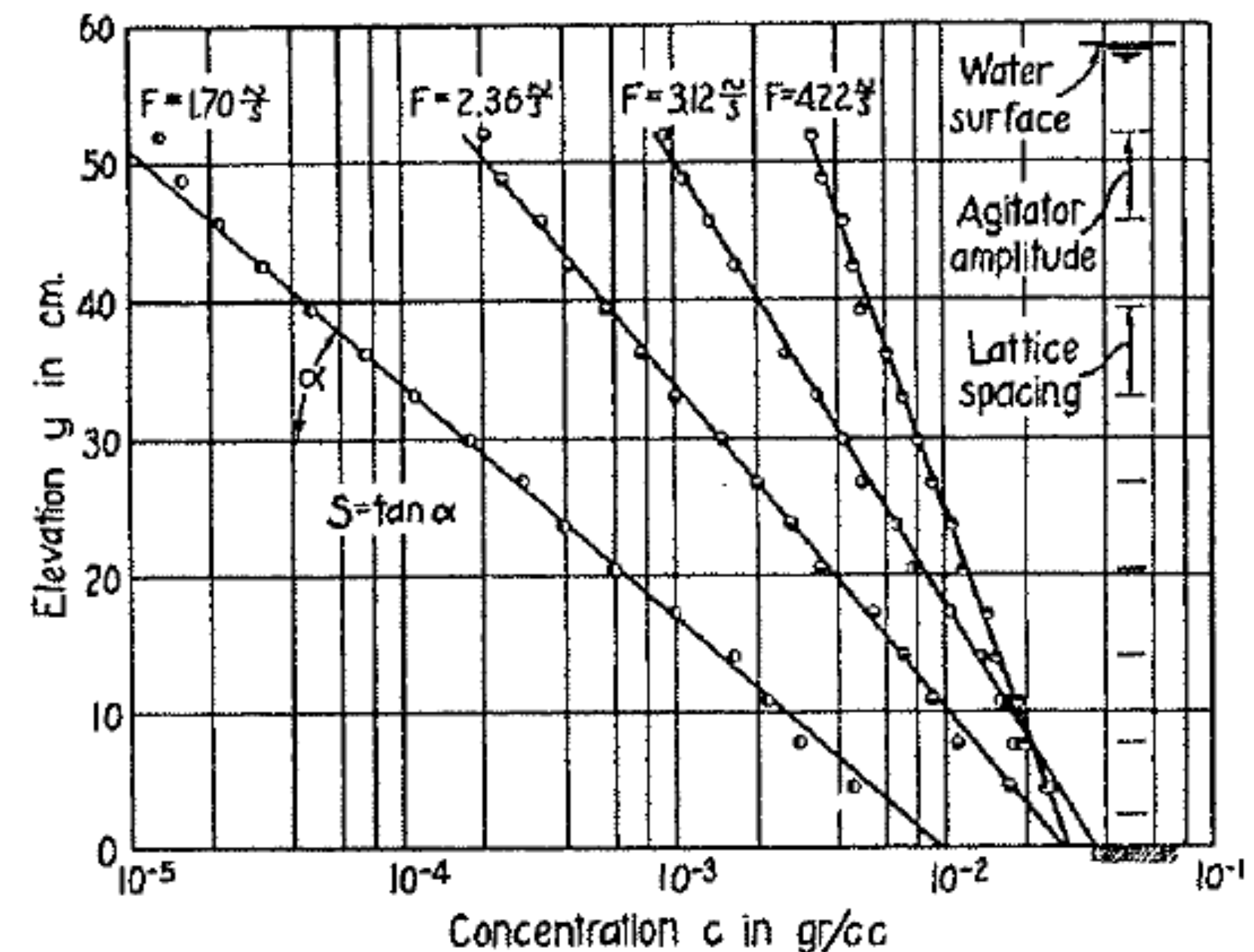
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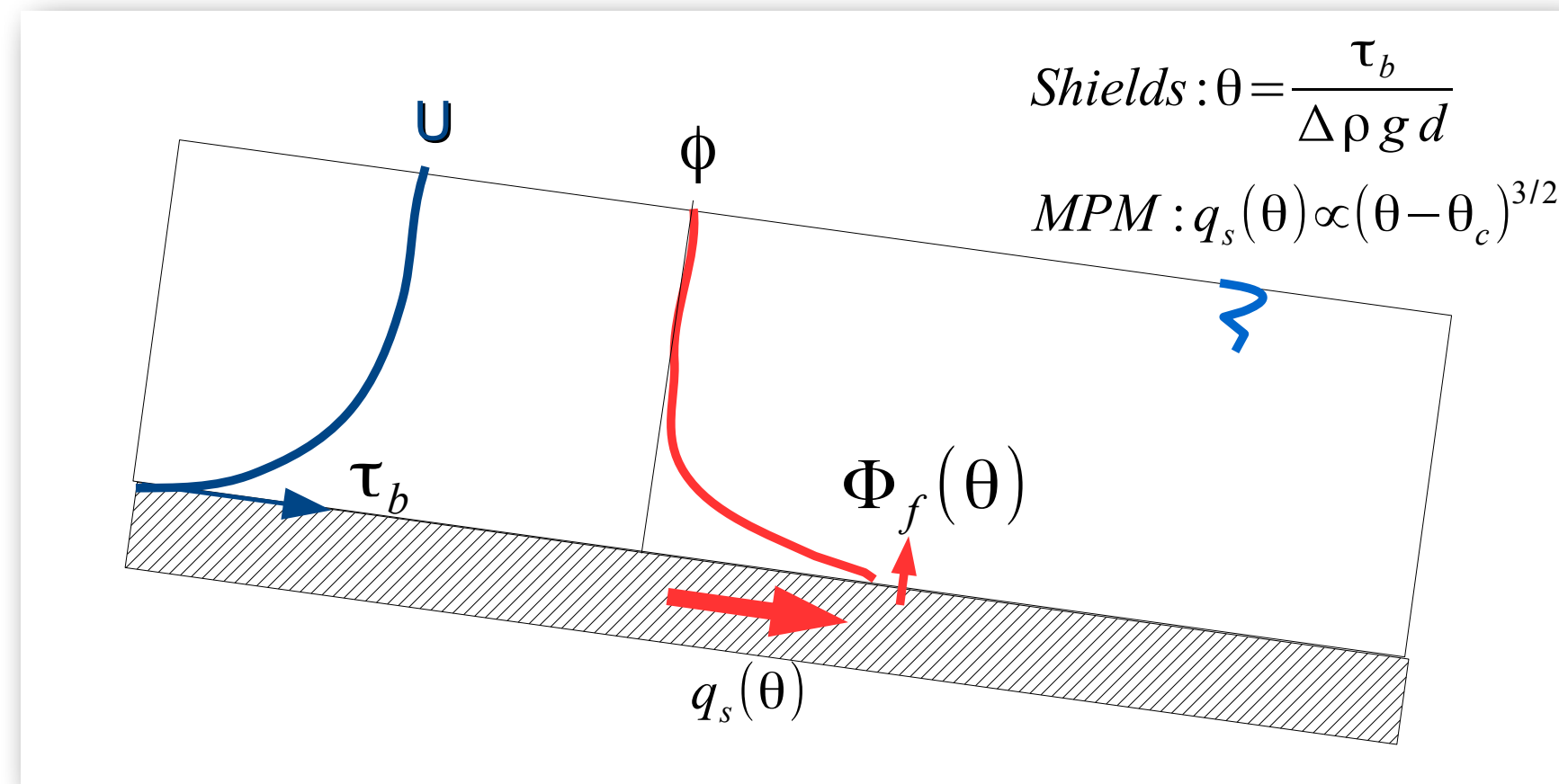
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OPEN-CHANNEL FLOWS



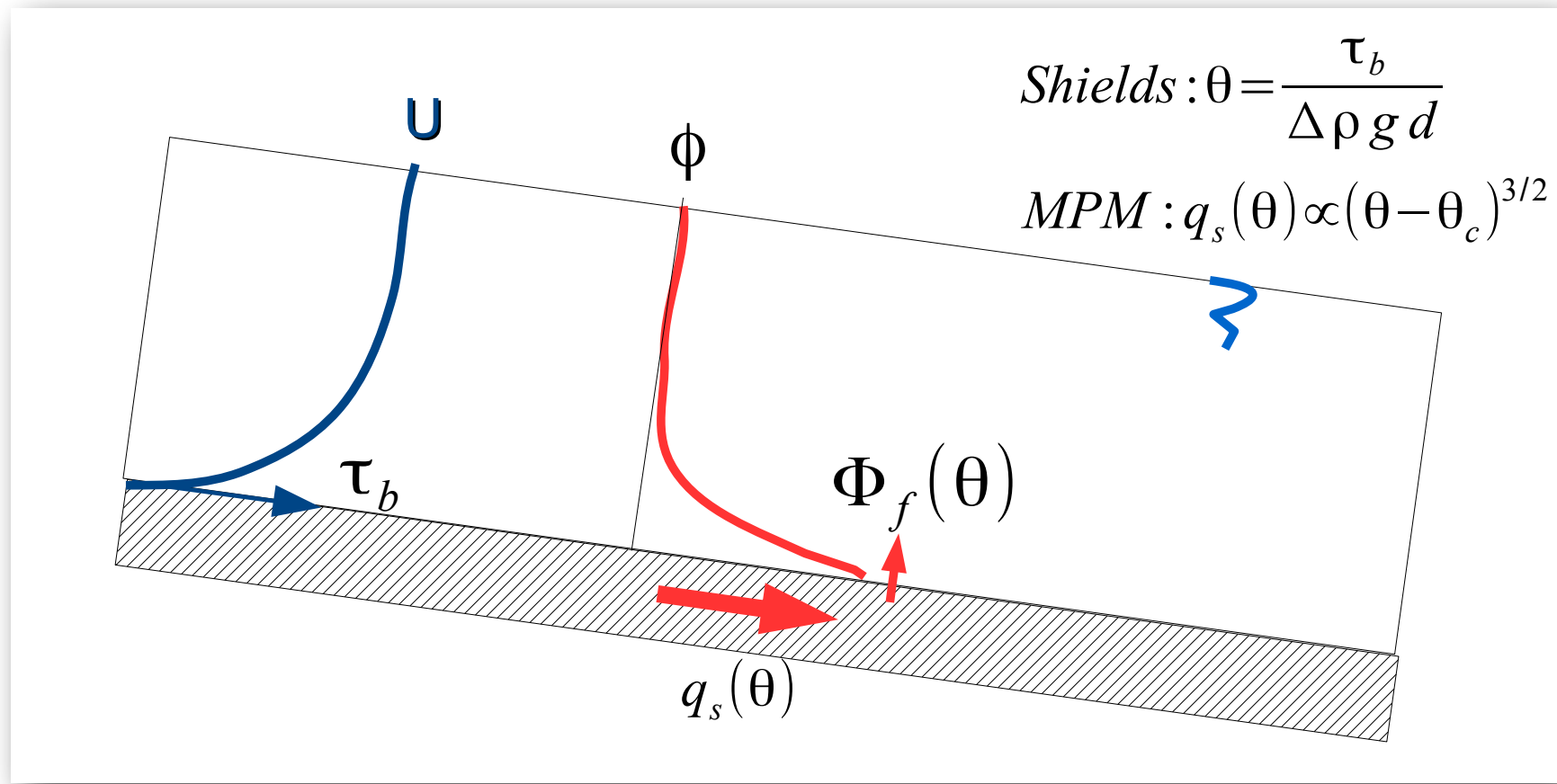
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Concentration diffusivity: $\varepsilon_s = \frac{\nu^t}{S_c}$

Rouse profile: $\frac{c}{c_a} = \left[\frac{z}{H_f - z} \frac{H_f - a}{a} \right]^{-Ro}$

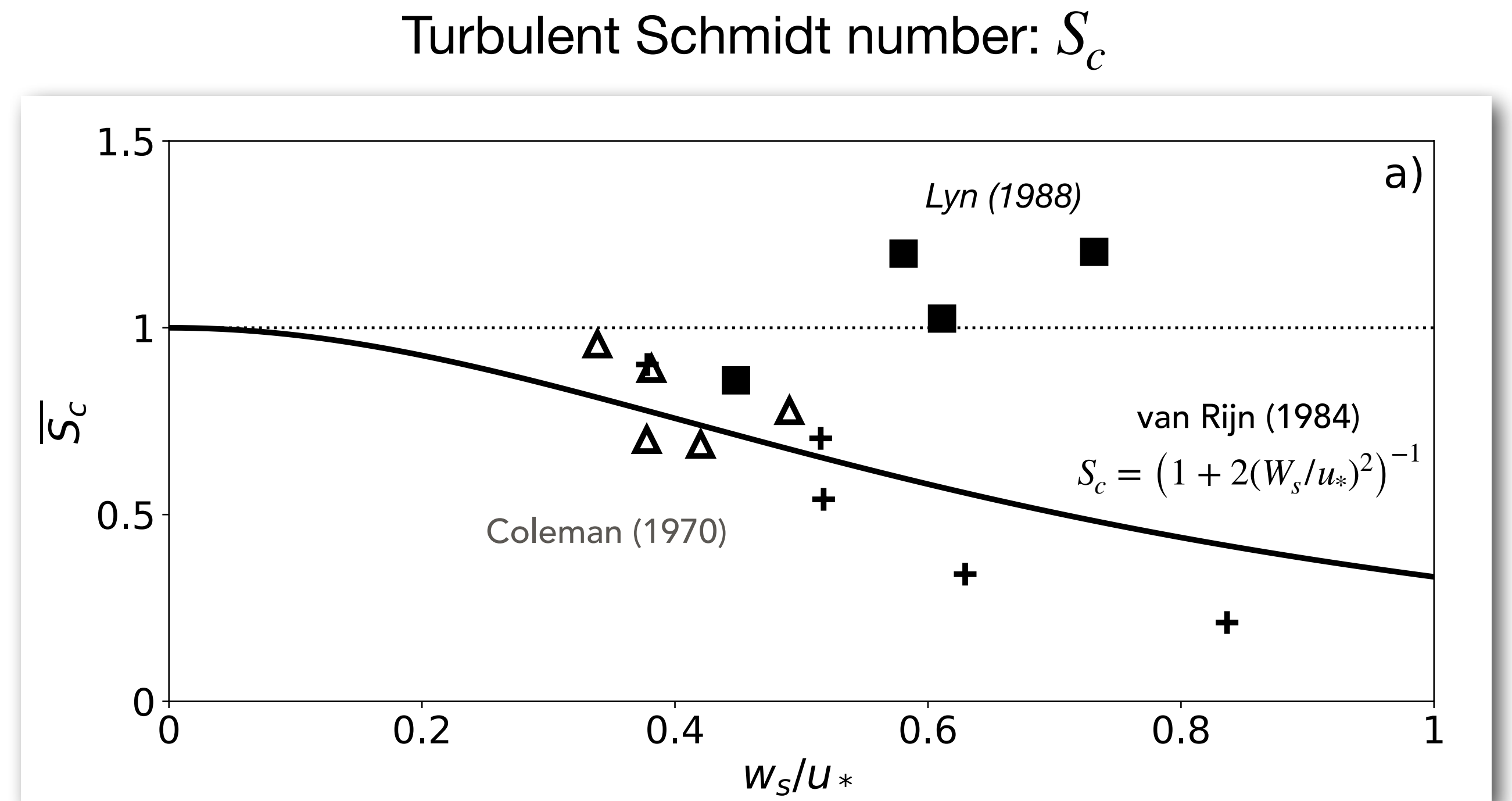
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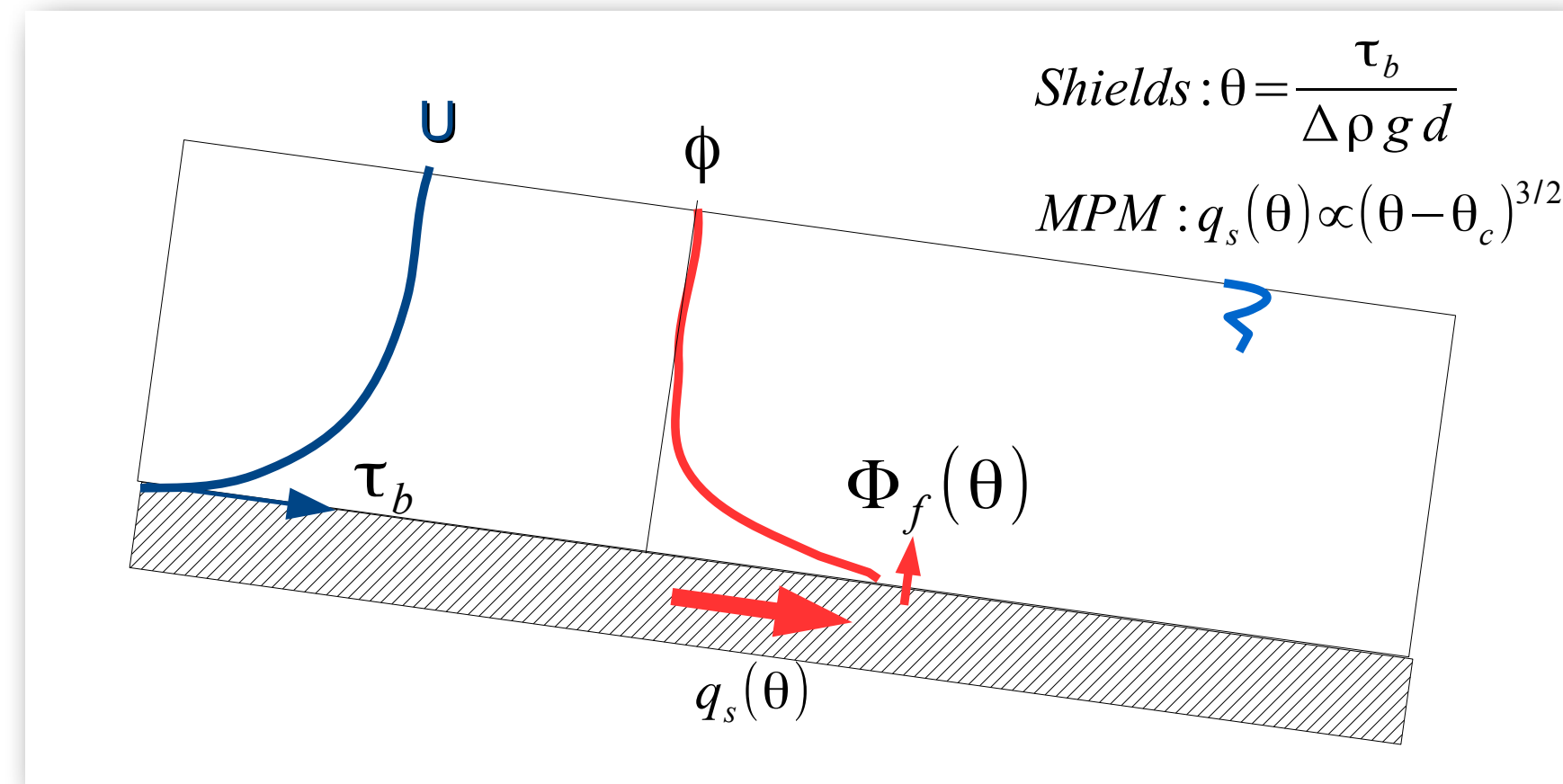
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Data from open-channel flow experiments collected by Lyn (2008)

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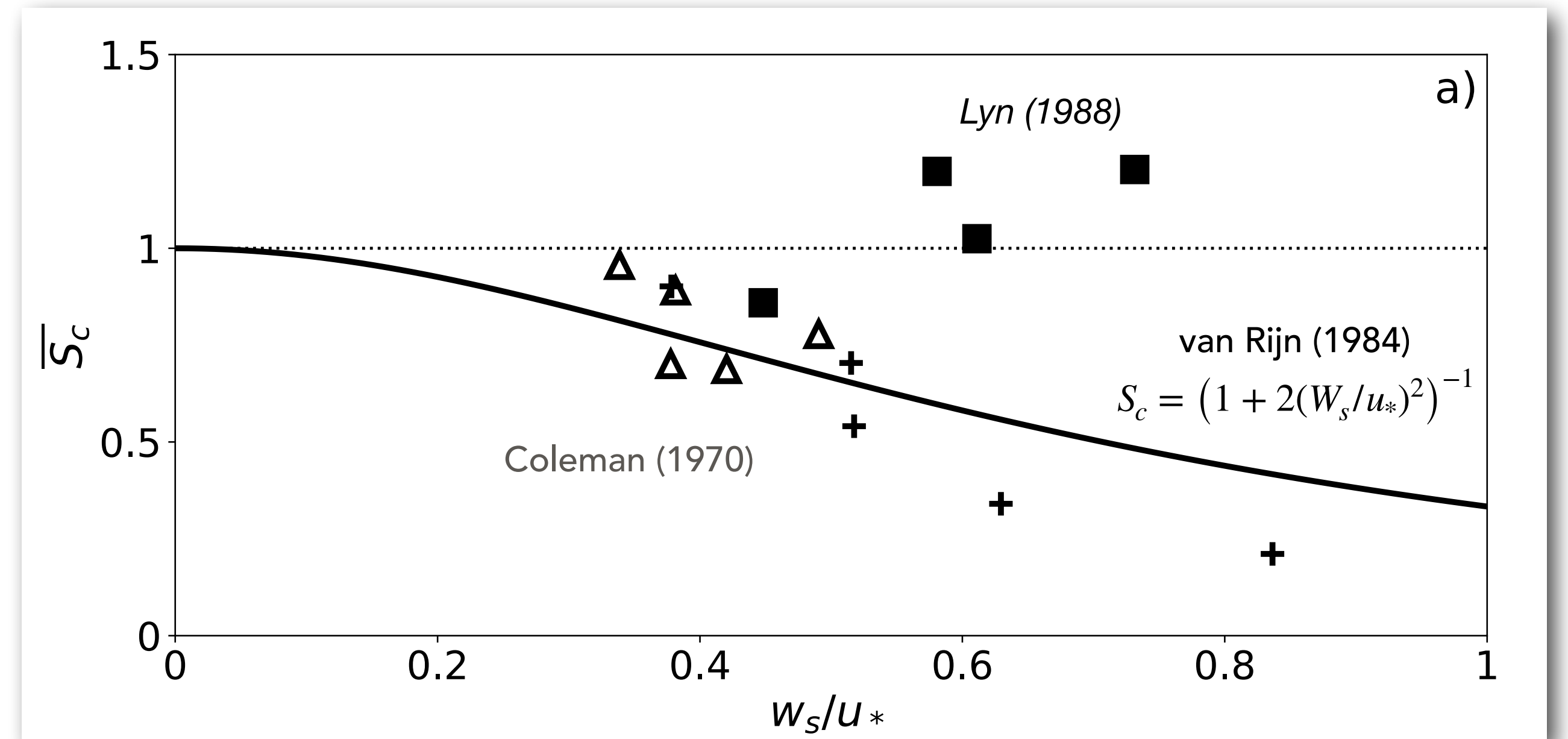
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Turbulent Schmidt number: S_c



Data from open-channel flow experiments collected by Lyn (2008)

What could explain $S_c < 1$?

Eulerian-Eulerian two-phase flow equations

Local mass & momentum conservation for a fluid-particle mixture

$$\nabla \cdot \vec{u} = 0 \quad \text{and} \quad \frac{d\rho\vec{u}}{dt} + \nabla \cdot (\rho\vec{u} \otimes \vec{u}) = \nabla \cdot \bar{\bar{\sigma}} + \rho\vec{g}$$

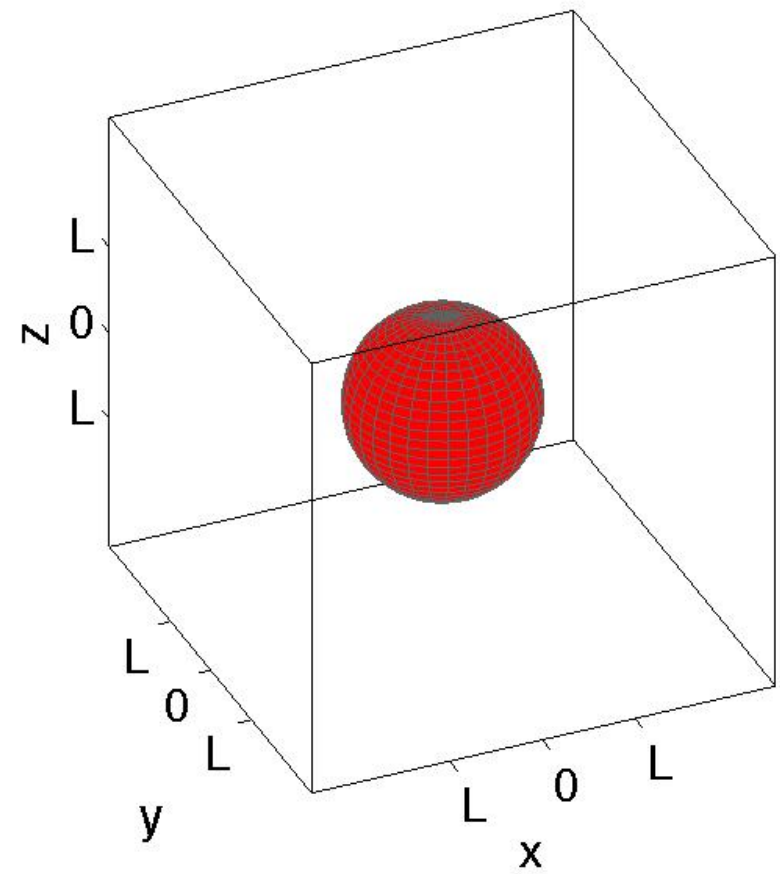
Local spatial averaging

Jackson (2000)

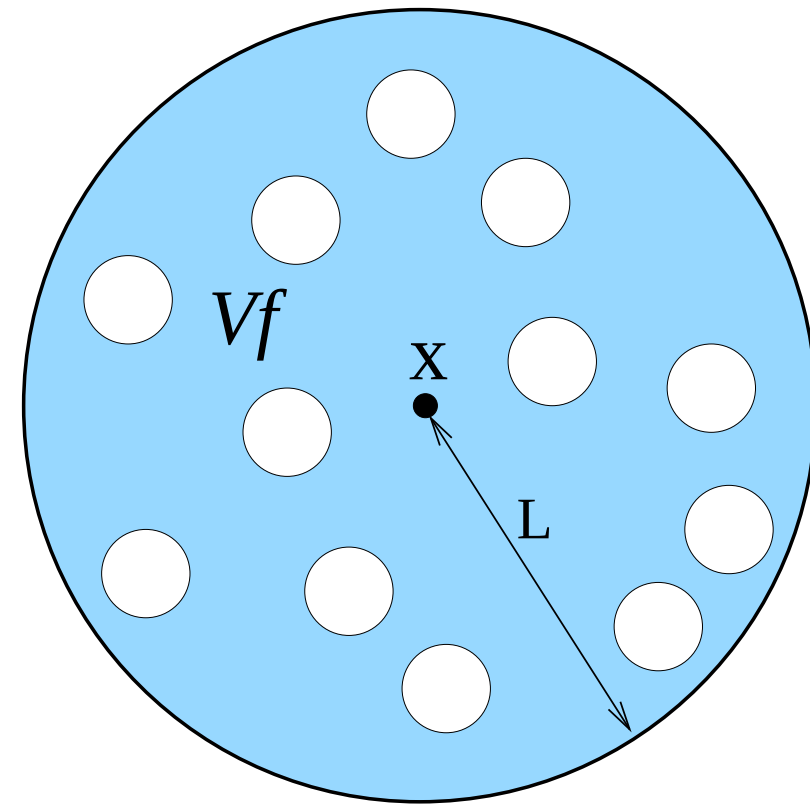
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G_L is a 3D door function



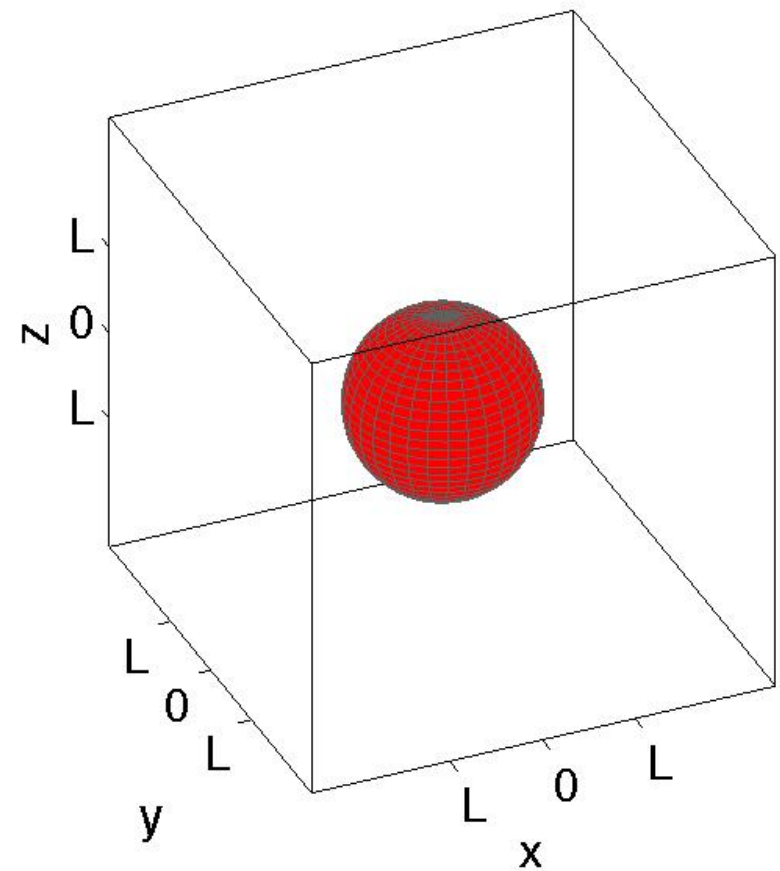
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$$\langle f \rangle^f(\vec{x}, t) = \frac{1}{1 - \phi(\vec{x}, t)} \int_{V_f(t)} f(\vec{y}, t) G_L(|\vec{x} - \vec{y}|) dV_y$$

$$L_{flow} \gg L \gg d$$

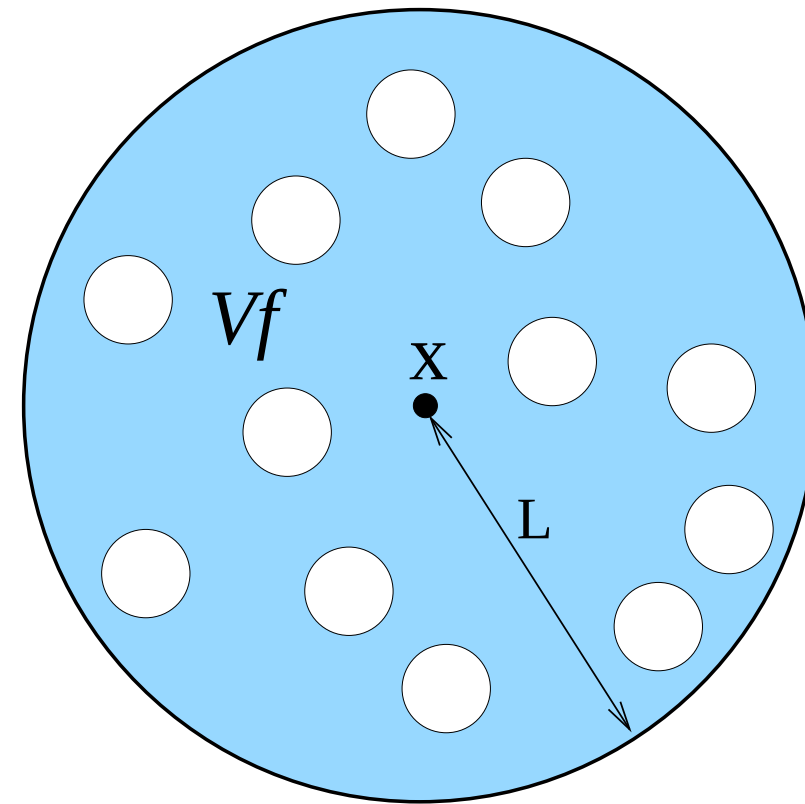
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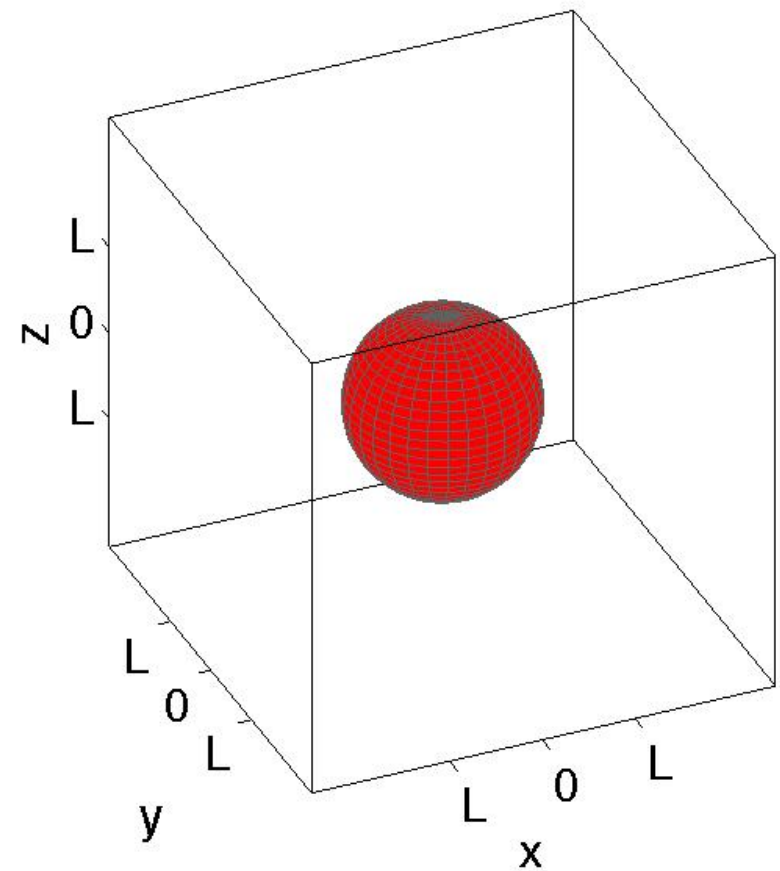
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$$\rho^f \left[\frac{\partial(1 - \phi) \langle \vec{u} \rangle^f}{\partial t} + \nabla \cdot ((1 - \phi) \langle \vec{u} \rangle^f \otimes \langle \vec{u} \rangle^f) \right] = \nabla \cdot \bar{\bar{\sigma}}^f - n \vec{f} + (1 - \phi) \rho^f \vec{g}$$

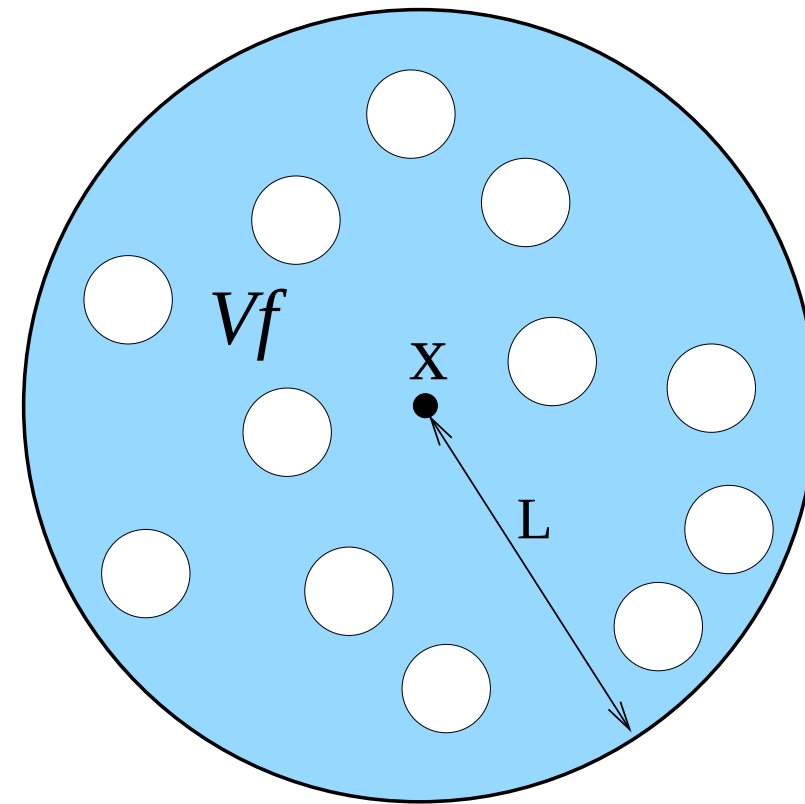
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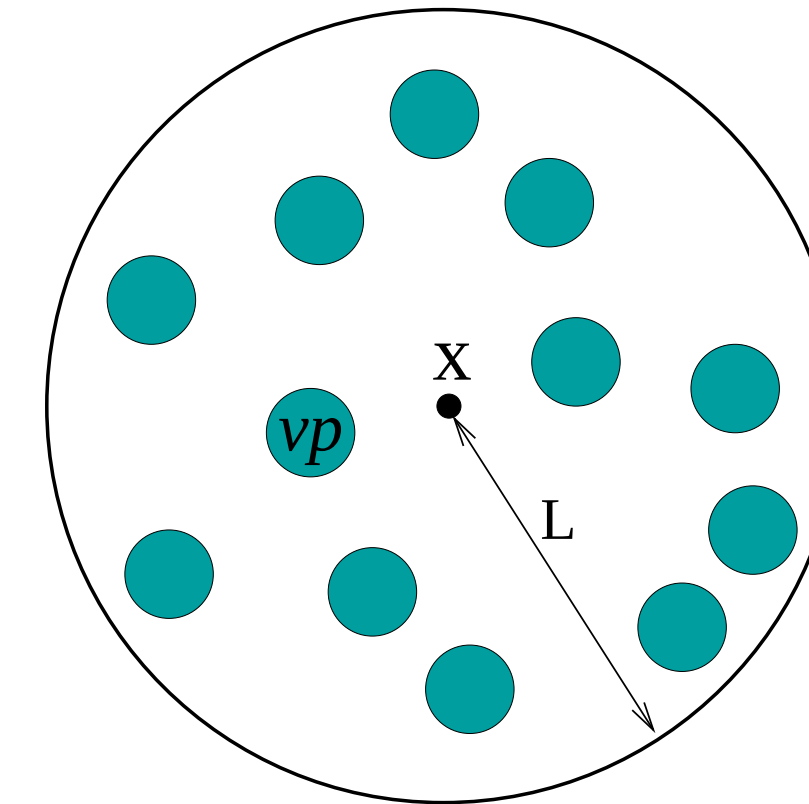
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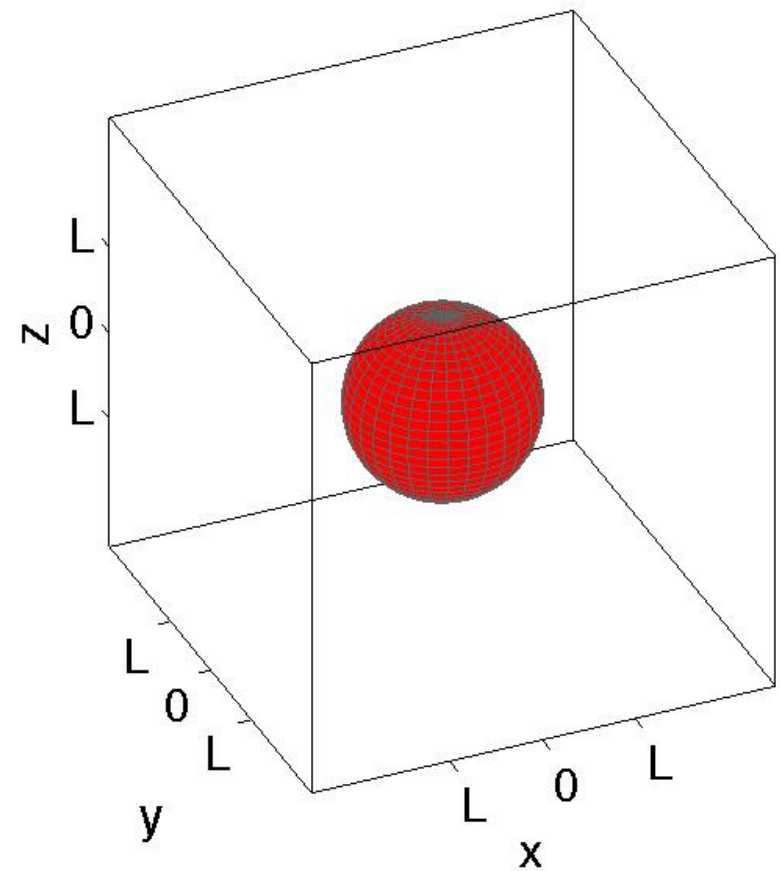
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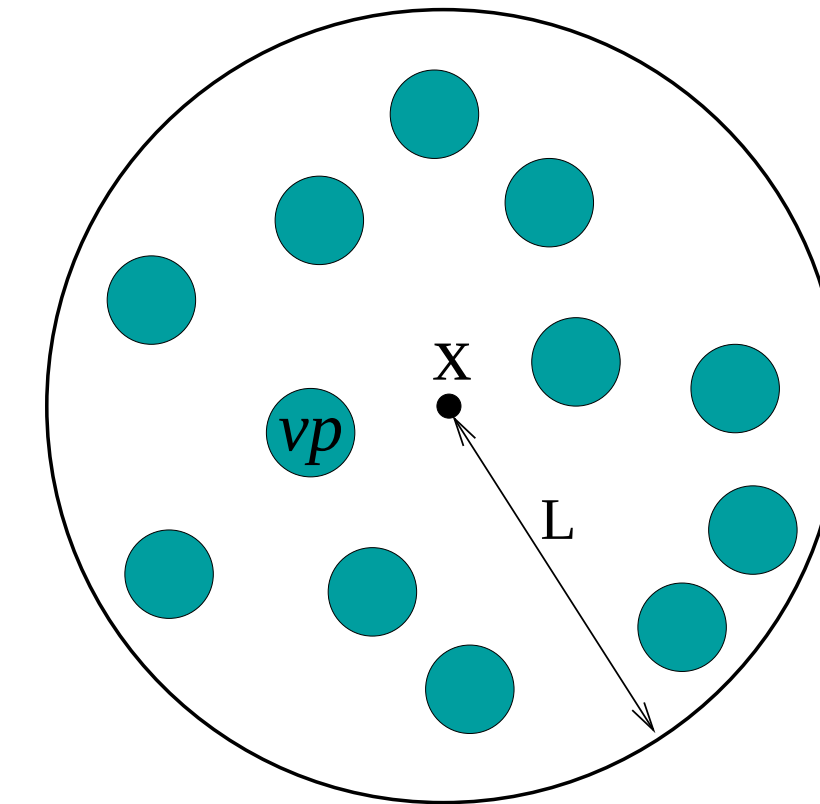
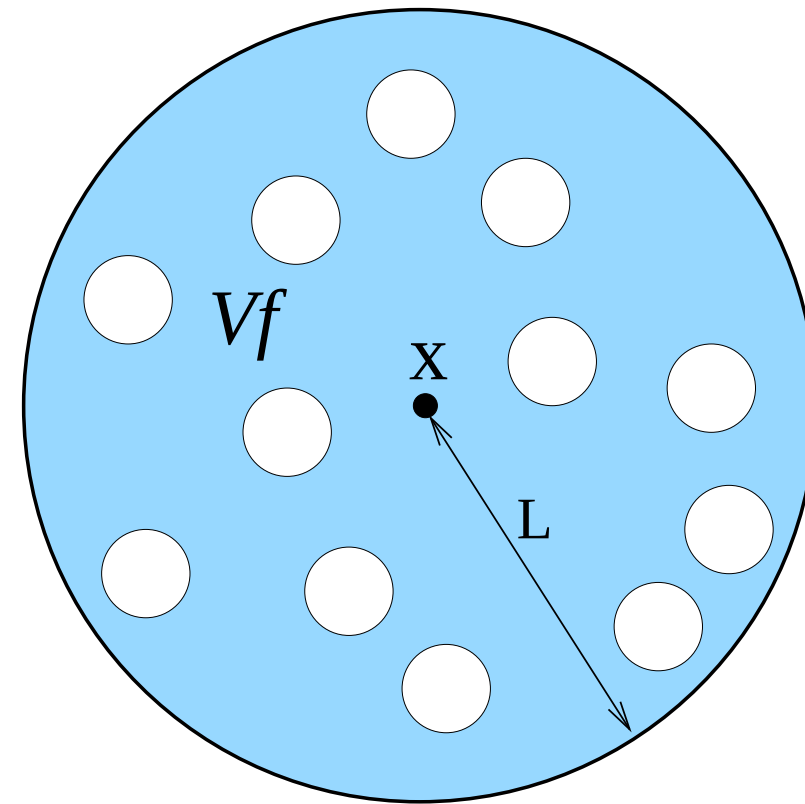
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Solid phase mass and momentum equations

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Favre-averaged two-phase flow equations

- Ensemble averaging: $\langle \phi \rangle = \lim_{N \rightarrow \infty} \sum_{k=1}^N \phi_k$ Favre-averaged velocity $\overline{u^f} = \frac{\langle (1 - \phi) \vec{u}^f \rangle}{1 - \langle \phi \rangle}$ $\overline{u^{pp}} = \frac{\langle \phi \vec{u}^p \rangle}{\langle \phi \rangle}$
- Favre-averaged two-phase flow equations:

$$\begin{aligned} \frac{\partial \langle \epsilon \rangle}{\partial t} + \nabla \left(\langle \epsilon \rangle \overline{u^f} \right) &= 0 \\ \rho^f \left[\frac{\partial \langle \epsilon \rangle \overline{u^f}}{\partial t} + \nabla \left(\langle \epsilon \rangle \overline{u^f} \otimes \overline{u^f} \right) \right] &= -\rho^f \nabla \left(\langle \epsilon \rangle \widetilde{\Delta u^f \otimes \Delta u^f} \right) - \overline{\nabla} p + \overline{\nabla} \cdot \overline{\tau^f} - \langle n \vec{f} \rangle + \langle \epsilon \rangle \rho^f \vec{g} \end{aligned}$$

Effective fluid stress
Viscous effects ~ negligible

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Reynolds-like stresses
Mixing length, k-ε or LES

Fluid-particle interactions
Drag + Buoyancy + dispersion

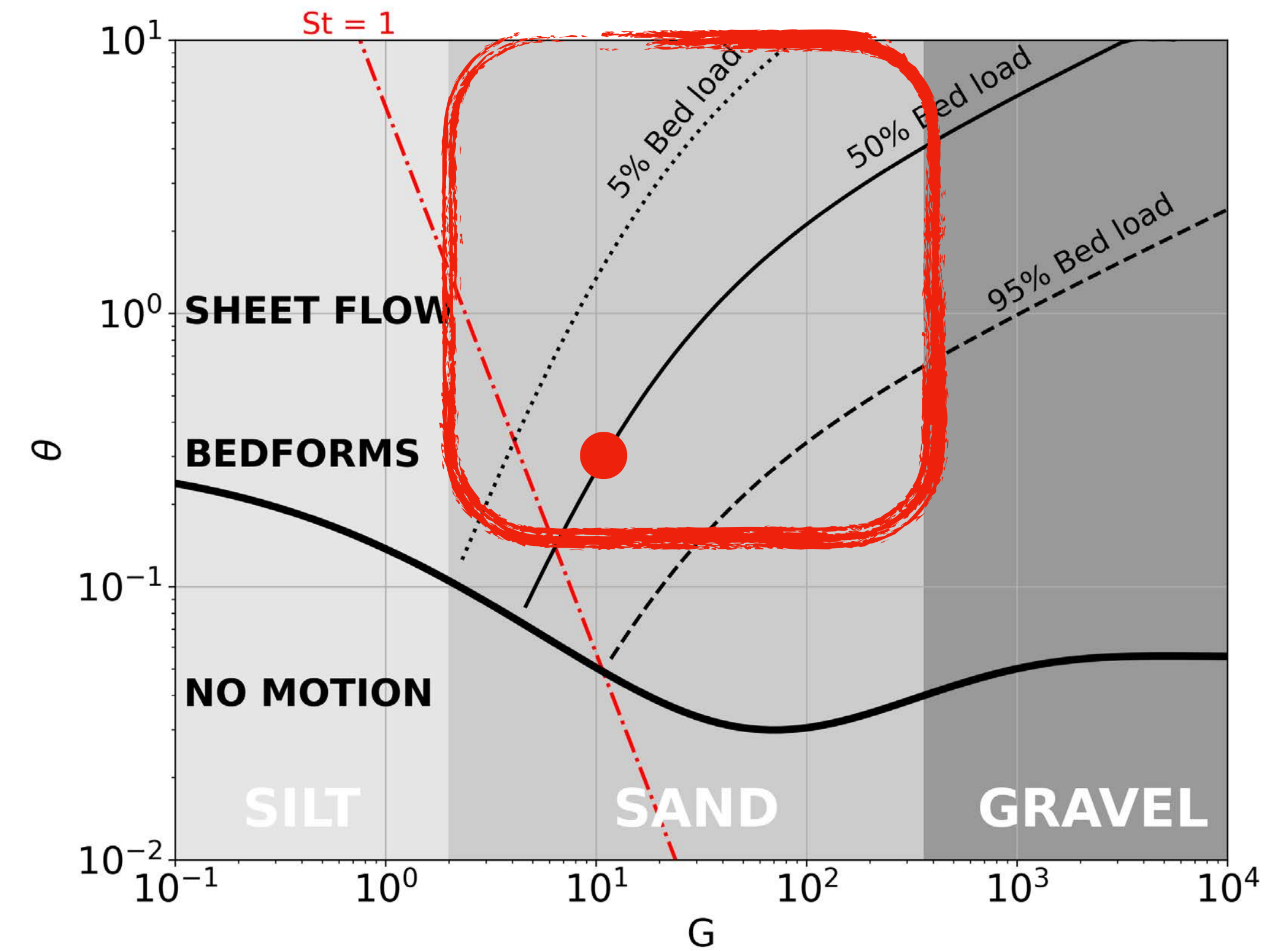
Granular stresses
μ(l) or Kinetic theory of granular flows

Two-fluid suspended-load simulation

Experiments with Glass Beads

Kiger & Pan (2002)

Parameters	Units	GB
U_b	$m.s^{-1}$	0.51
$u_\tau (\times 10^{-2})$	$m.s^{-1}$	2.99
h	m	0.02
ρ^s	$kg.m^{-3}$	2600
d_p	μm	195
$\phi_{tot} (\times 10^{-4})$	-	2.31
v_s/u_τ	-	0.87
Re_p	-	4.8
St	-	3.2
d_p/η	-	5.5

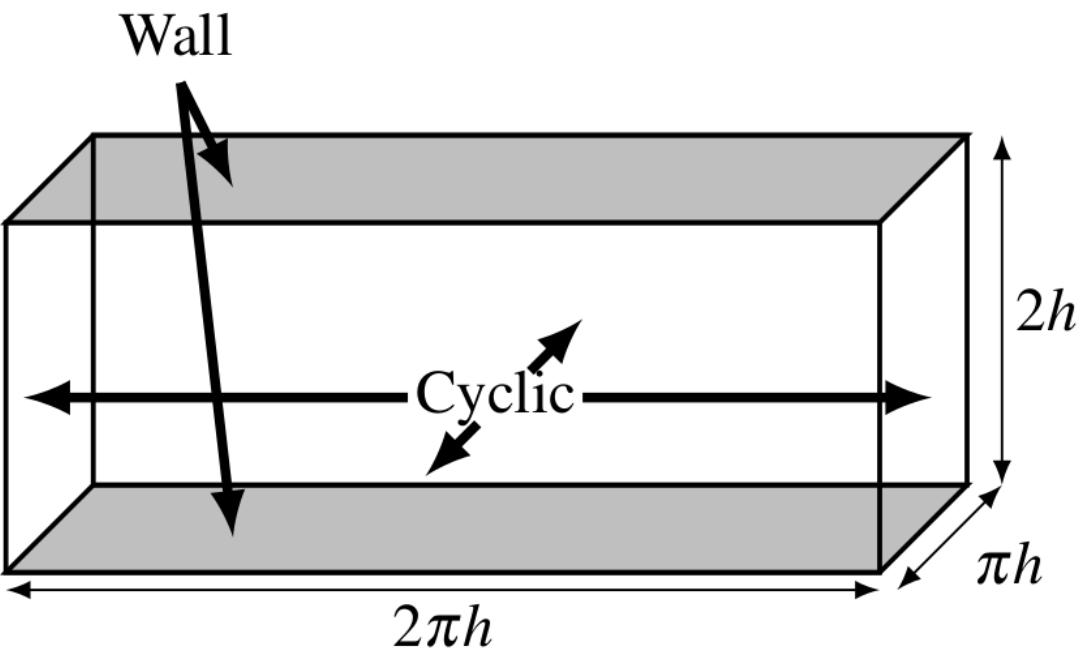
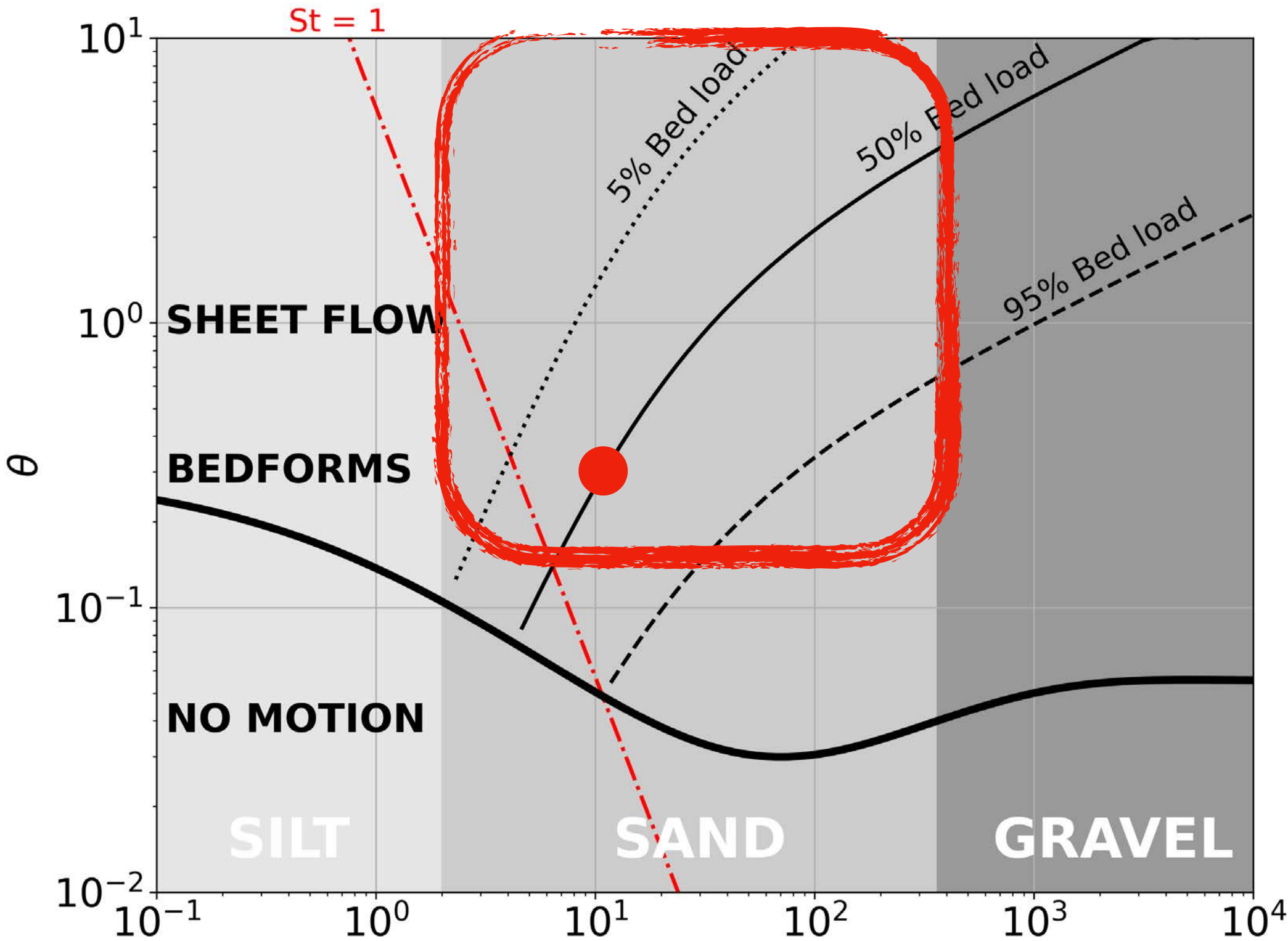


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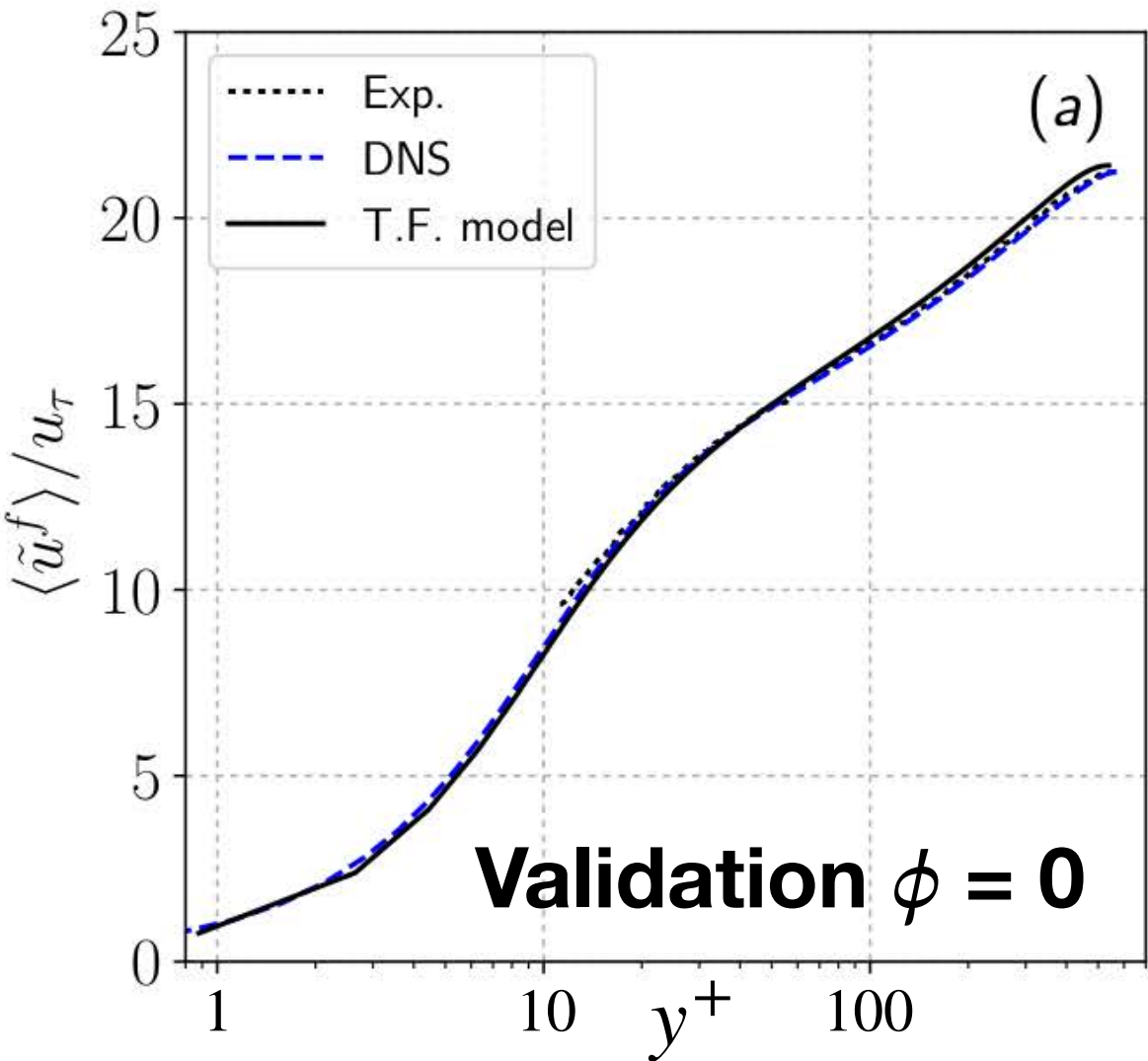


Numerical simulations: Mathieu et al. (JFM 2021)

- **2nd order finite volume** (openfoam/sedFOAM)
- **SGS model:** dynamic Lagrangian (Meneveau, 1996)

Mesh	Number of cells	Δ_x^+	Δ_z^+	Δ_y^+ (wall)
$314 \times 220 \times 160$	11,105,280	11	11	1

$$\frac{\Delta_x}{d_p} = \frac{\Delta_z}{d_p} \approx 2$$

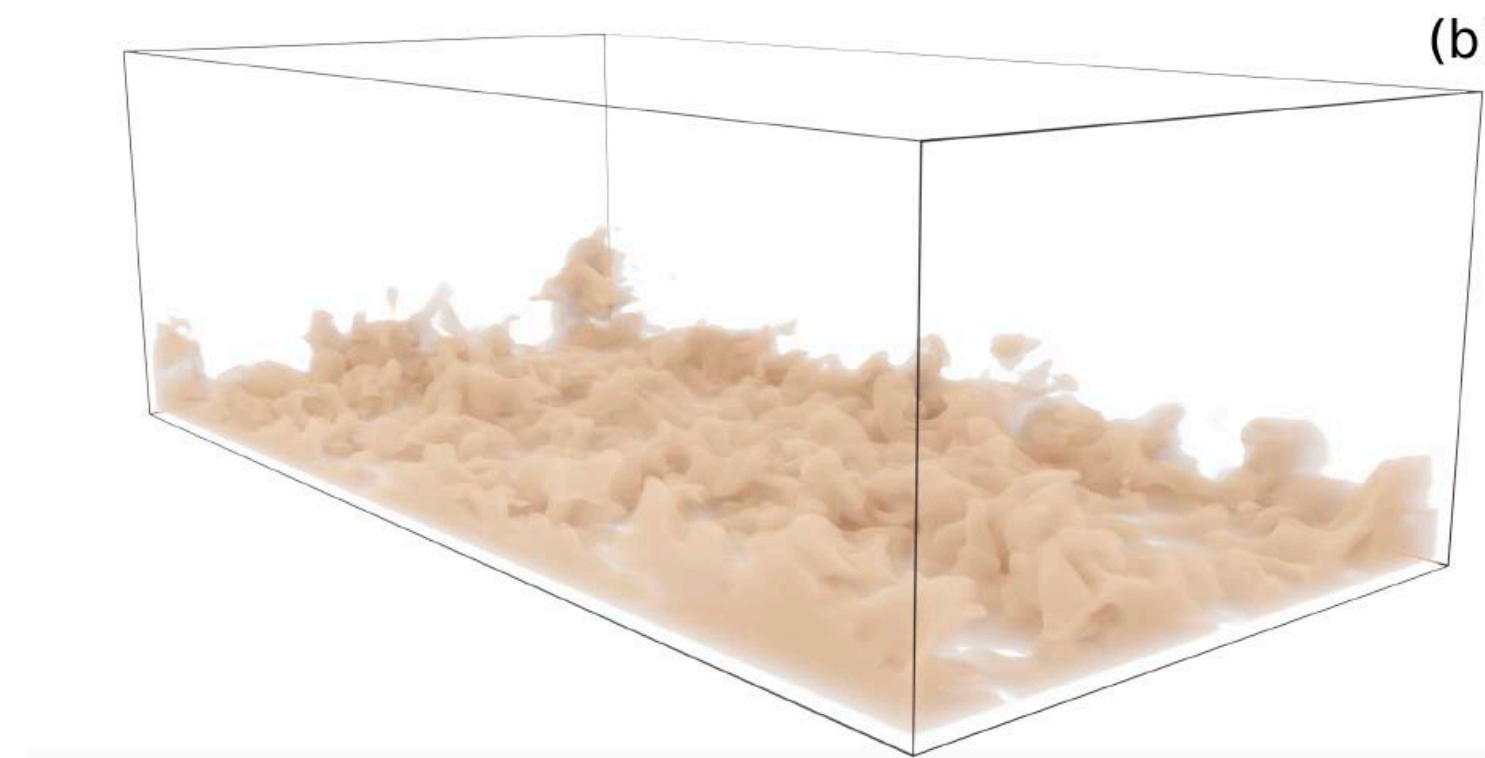


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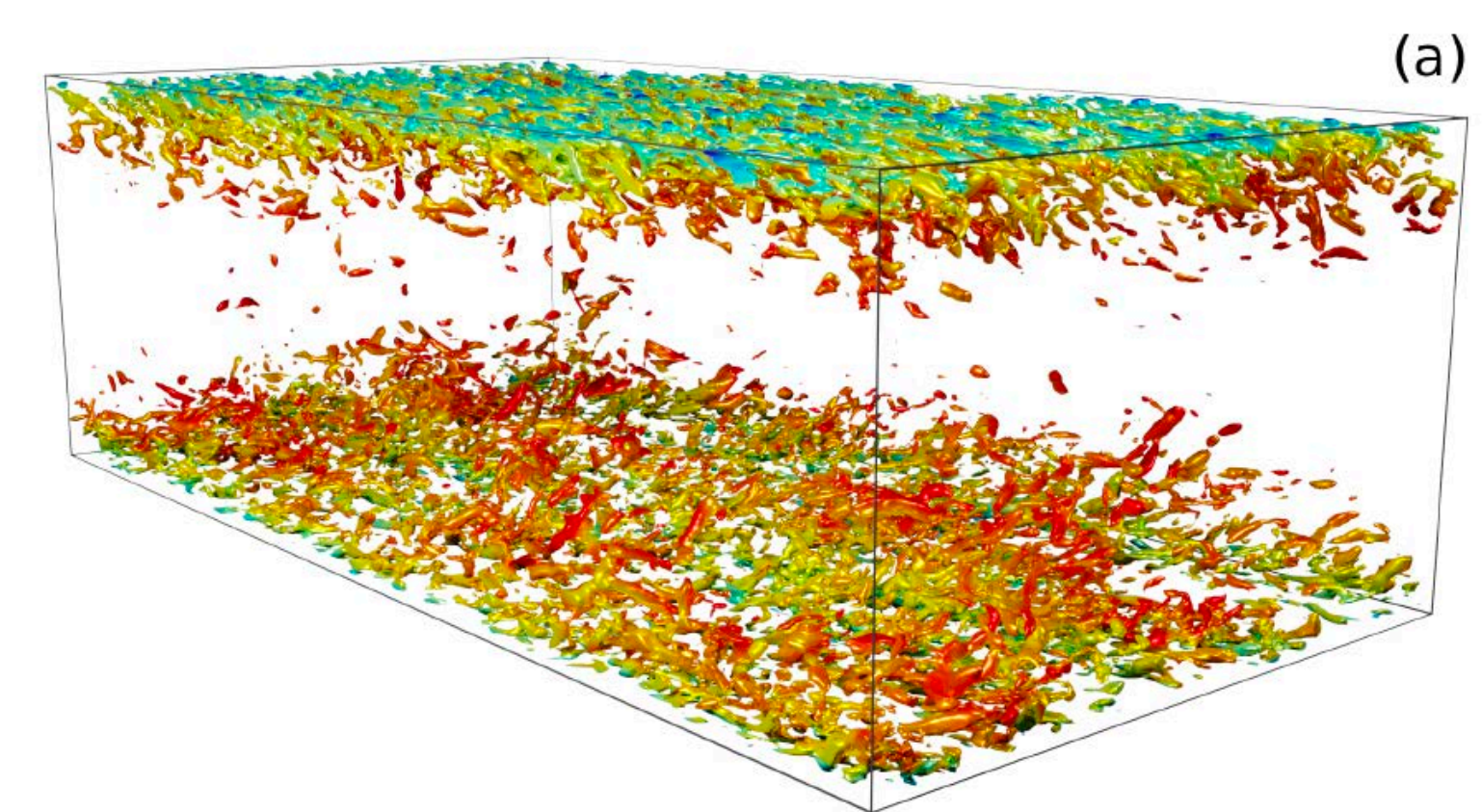
Glass beads experiments : Kiger & Pan (2002)

- $Re_b = 10^4$
- $Re_p = 4.8$
- $\bar{\phi} = 2.31 \times 10^{-4}$
- $Re_\tau = 560$
- $St = 3.2$
- $d_p/\eta = 5.5$

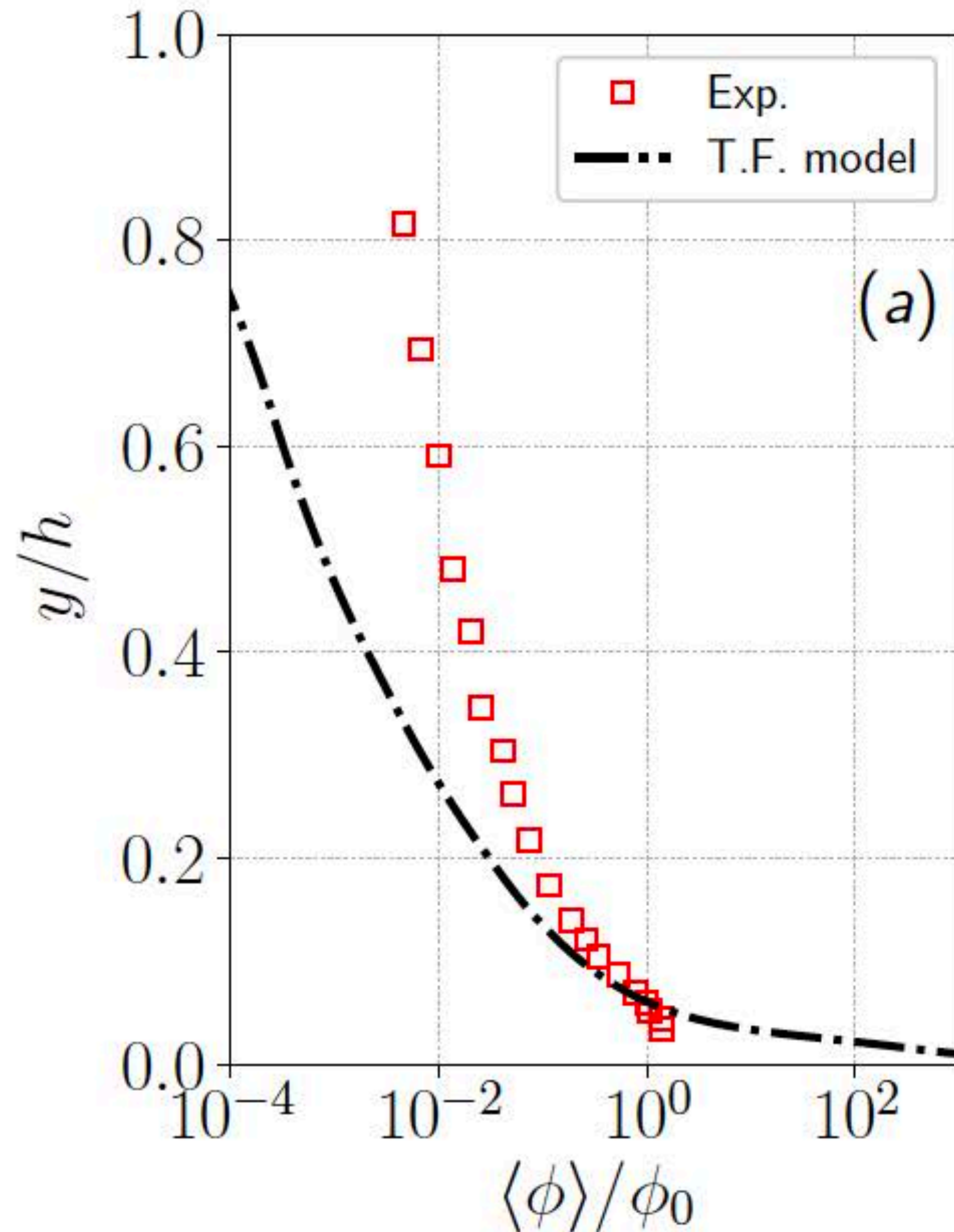
Contour of ϕ



Q criterion



Two-fluid suspended-load simulation



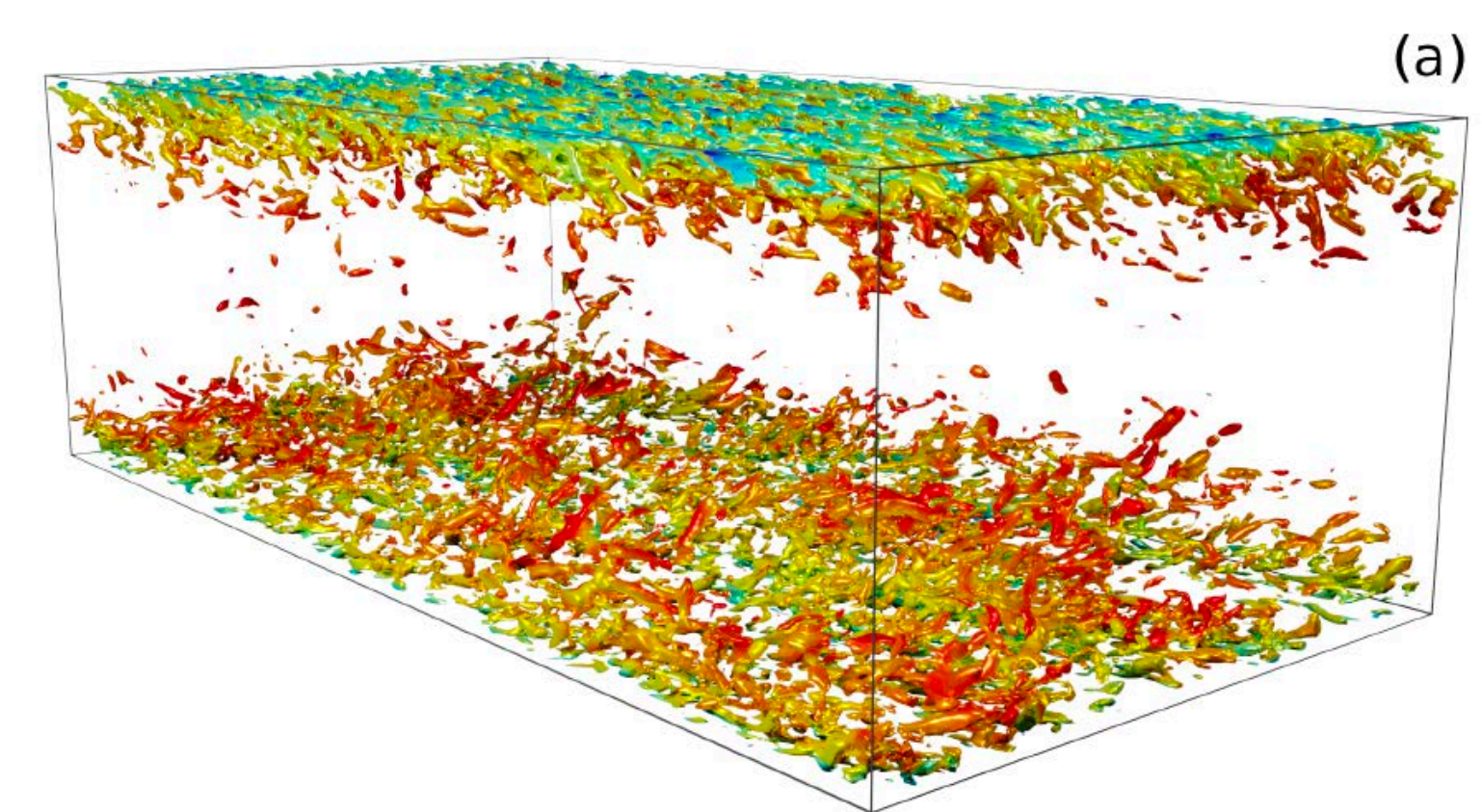
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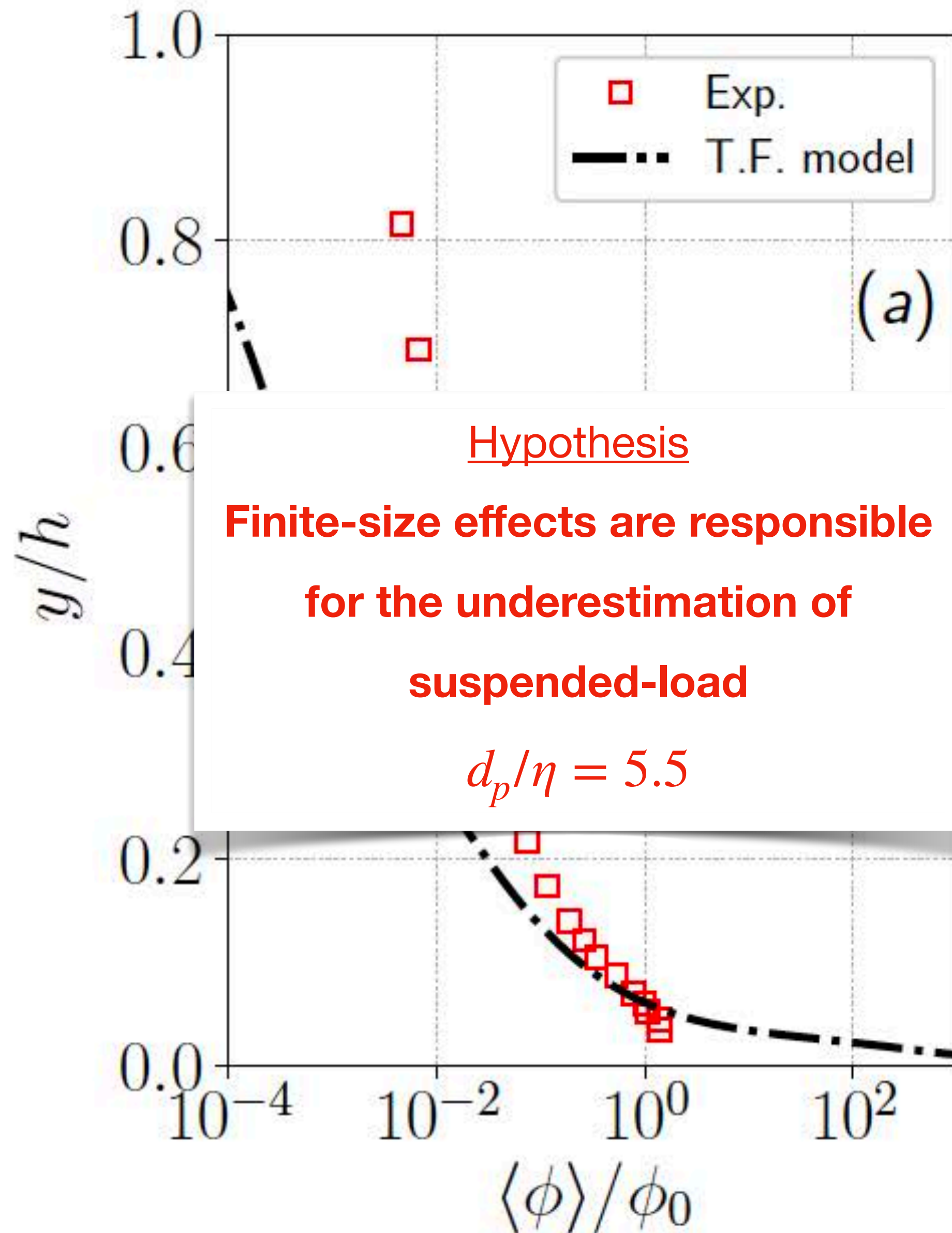
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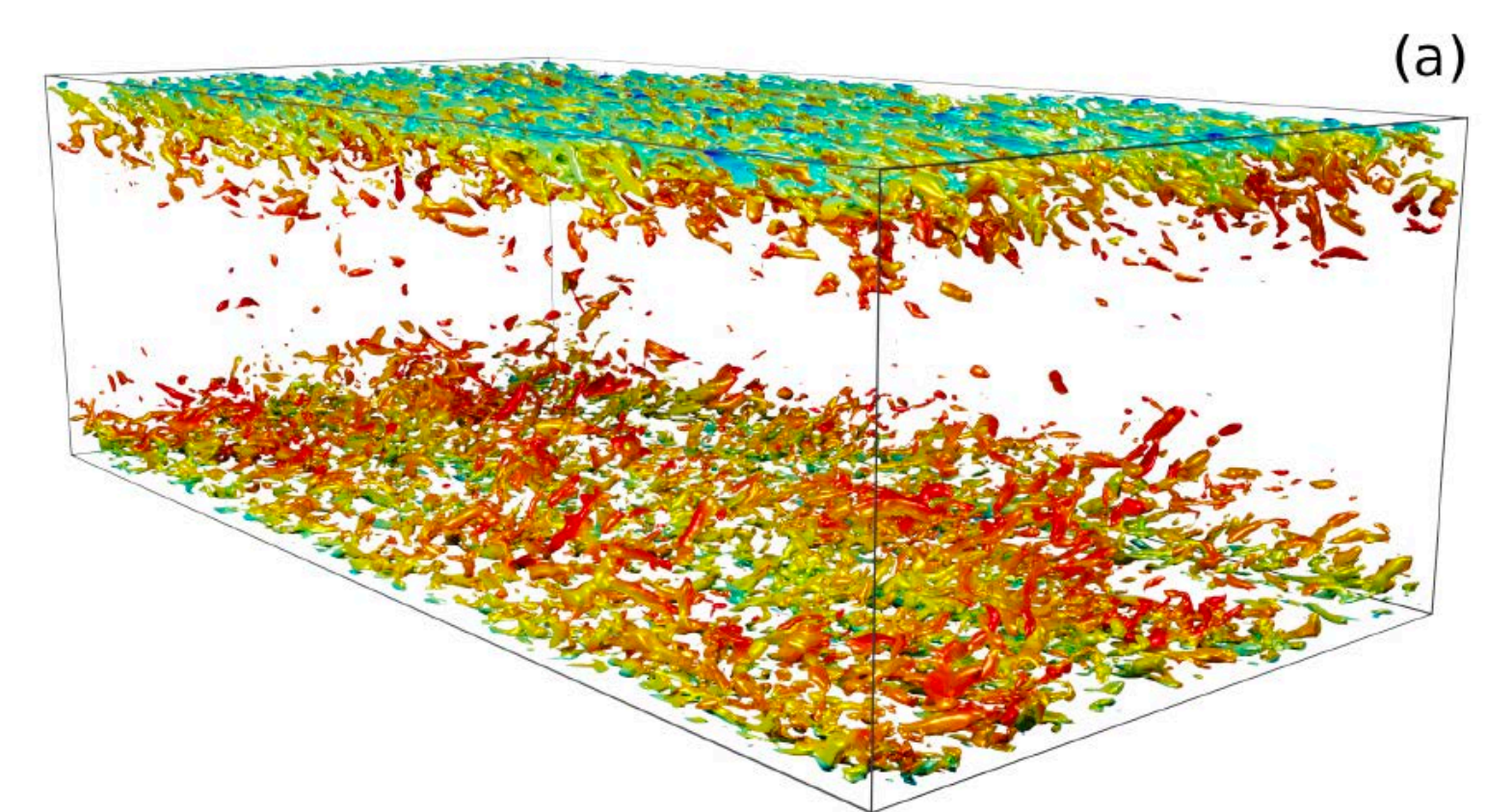
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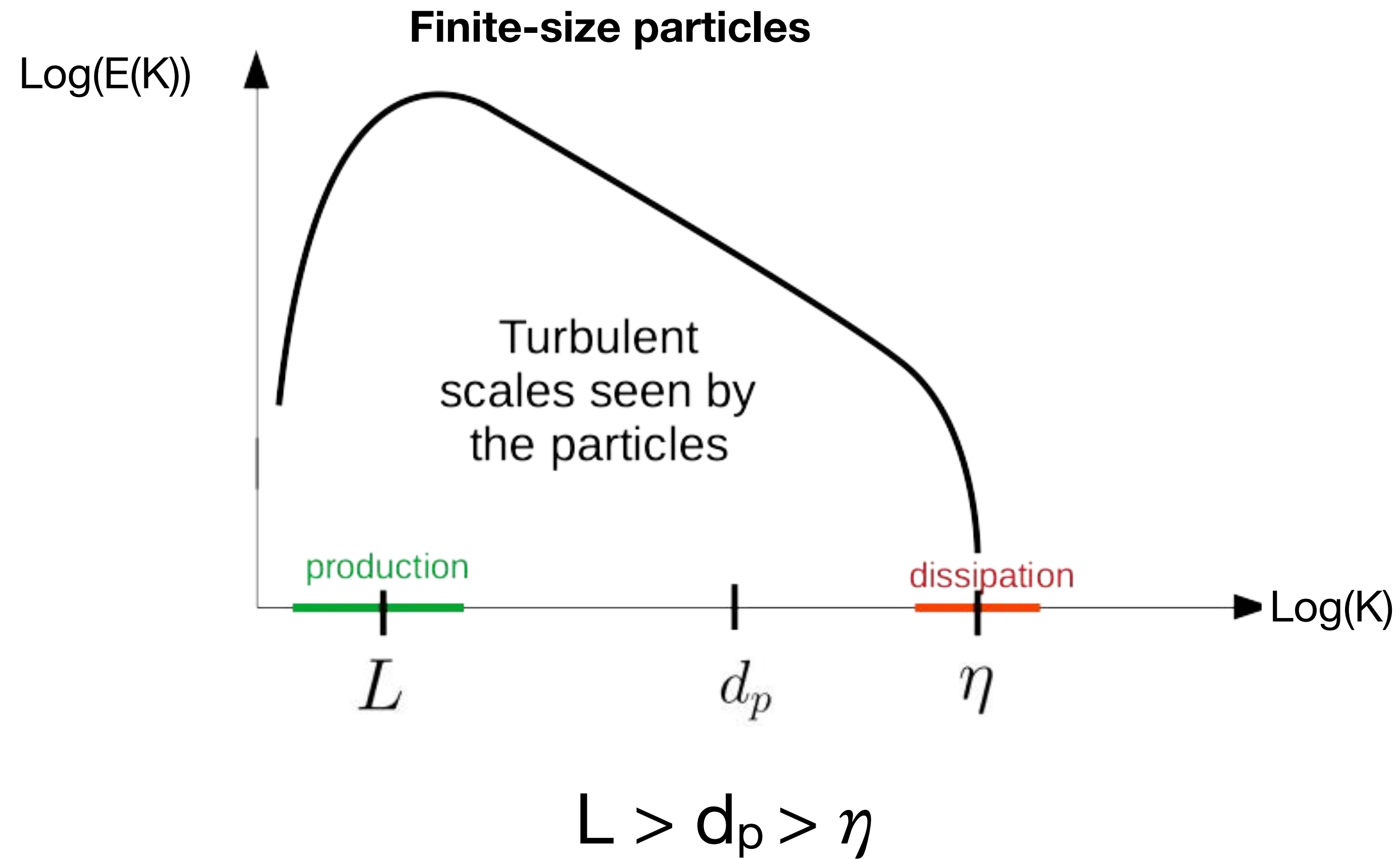
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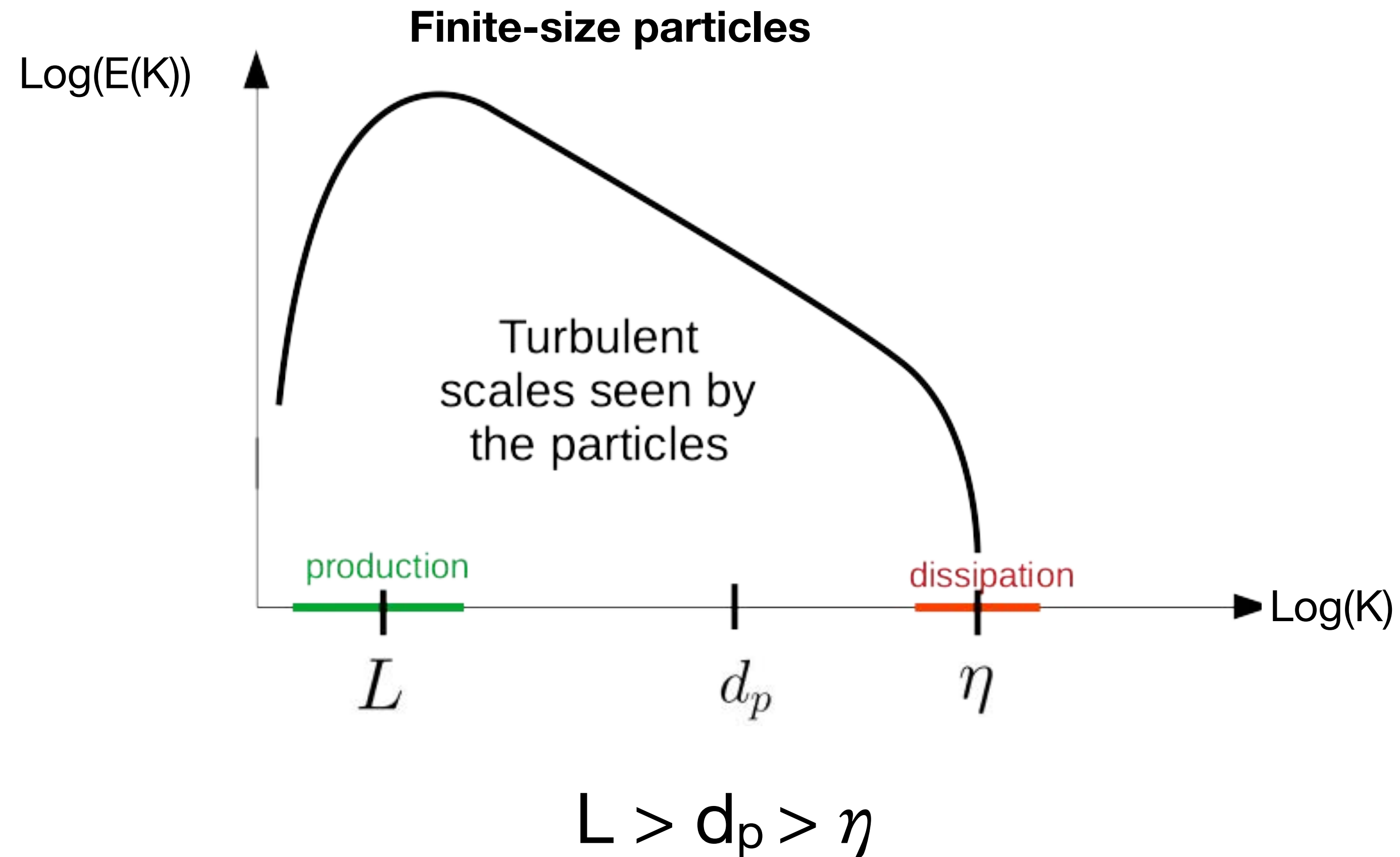
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Finite-size correction model for two-fluid LES



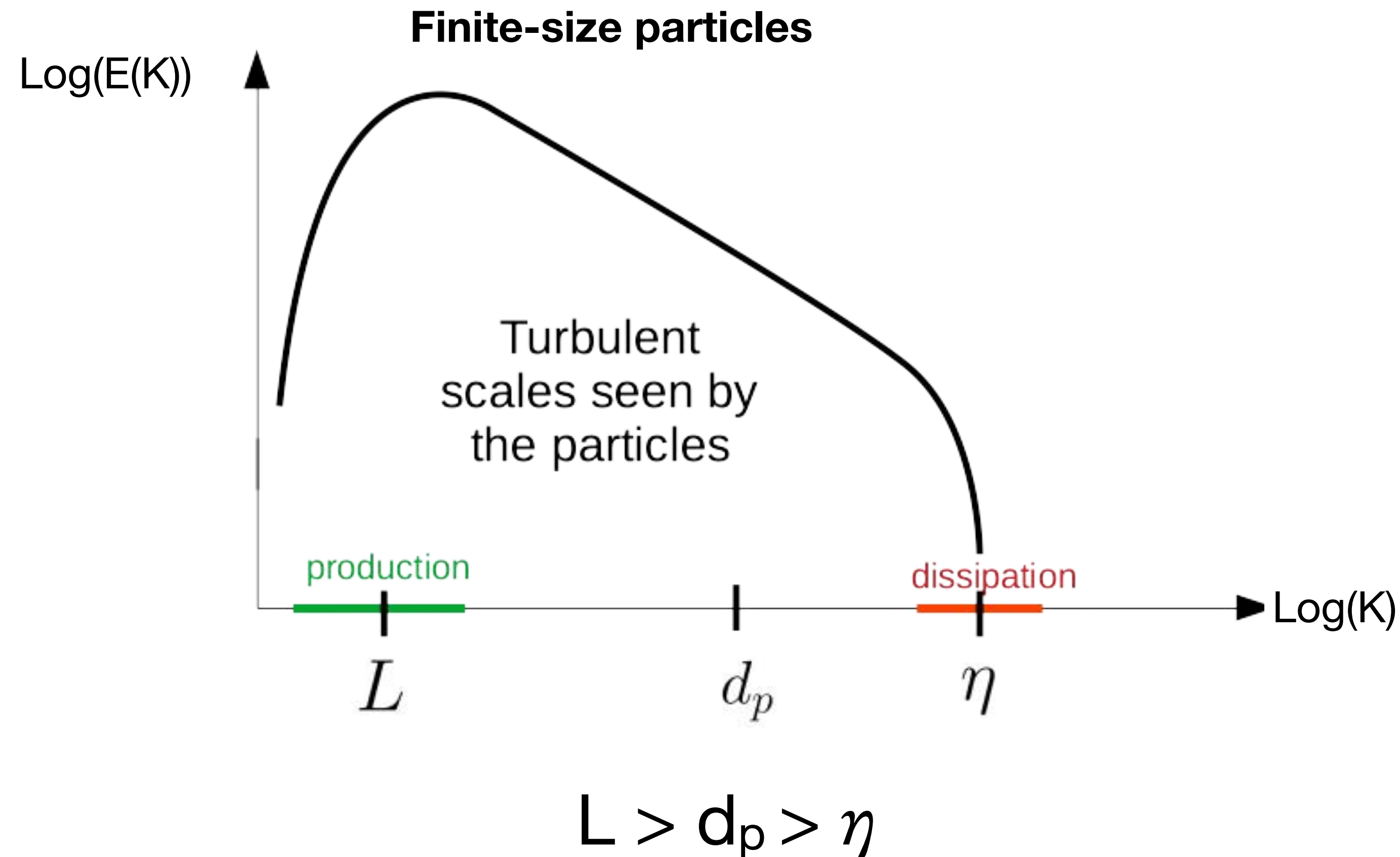
Finite-size correction model for two-fluid LES



Spatial scales forcing the particles dynamics are filtered

➡ *Only turbulent structures larger than particle size do contribute to their advection*

Finite-size correction model for two-fluid LES



Drag force:

$$\bar{D}_i = \frac{\rho^s \bar{\phi}}{\tilde{t}_s} \left(\tilde{u}_i^f - \tilde{u}_i^s \right)$$

Particle relaxation time:

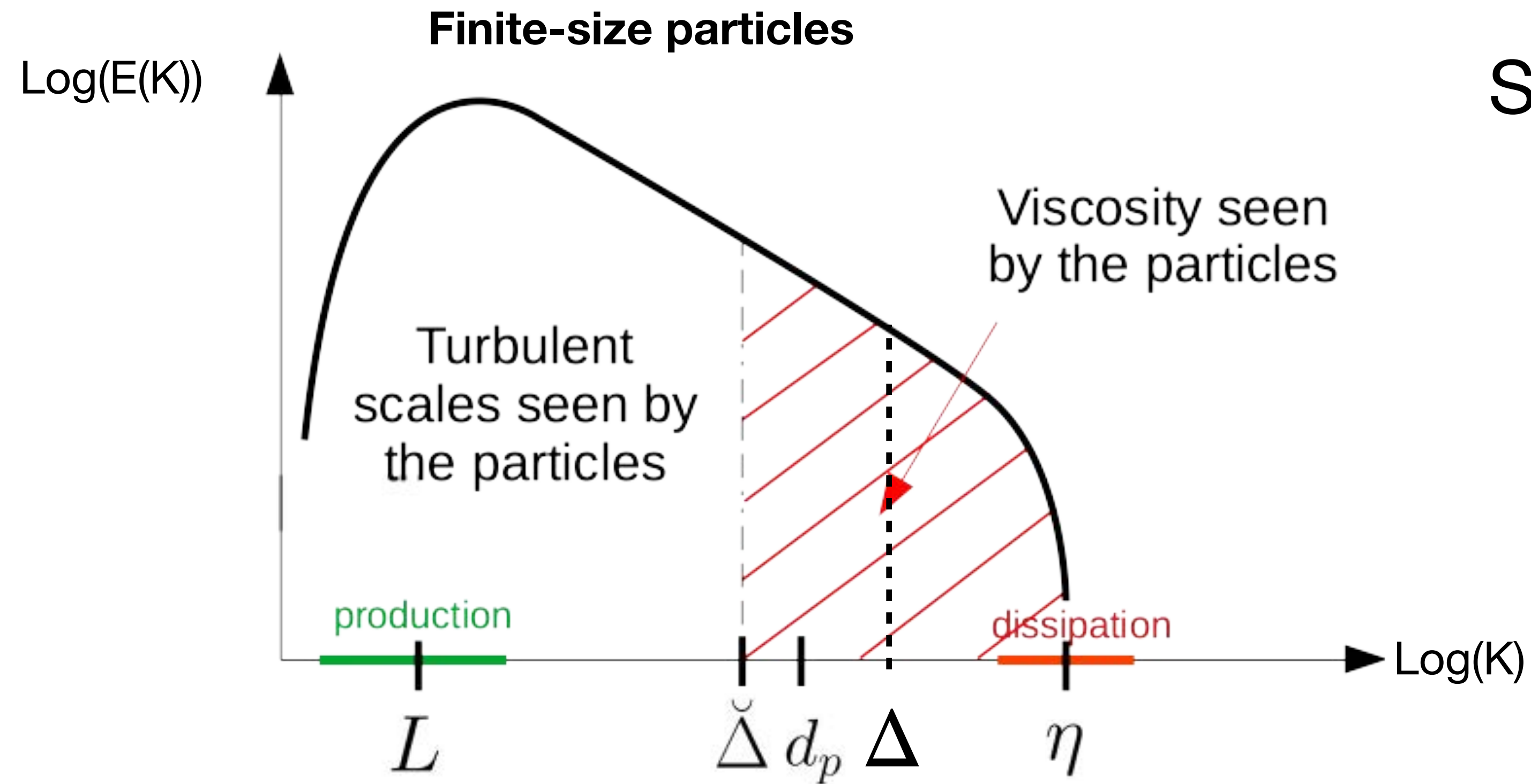
$$\tilde{t}_s = f \left(\|\tilde{u}_i^f - \tilde{u}_i^s\|, \nu^f \right)$$

Not valid anymore

Spatial scales forcing the particles dynamics are filtered

➡ *Only turbulent structures larger than particle size do contribute to their advection*

Finite-size correction model for two-fluid LES

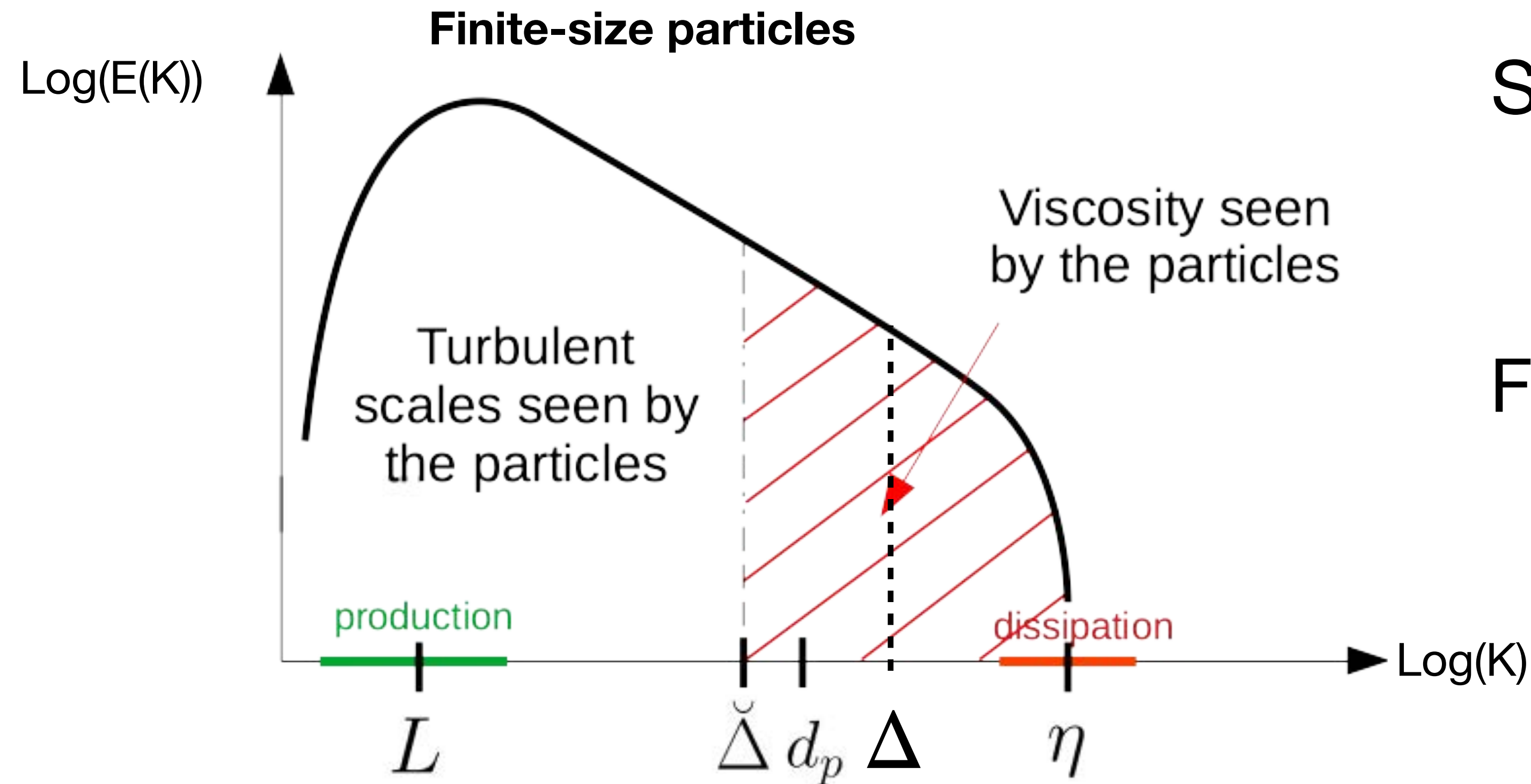


Spatial filter: $\tilde{\Delta} = 2d_p$

Kidanemariam et al. (NJP 2013)

Eddies smaller than d_p modify the particle response time by increasing the viscosity “seen” by the particles

Finite-size correction model for two-fluid LES



Spatial filter: $\tilde{\Delta} = 2d_p$

Kidanemariam et al. (NJP 2013)

Filtered drag force:

$$\bar{D}_i = \frac{\rho^s \bar{\phi}}{\tilde{t}_s} \left(\tilde{u}_i^f - \tilde{u}_i^s \right)$$

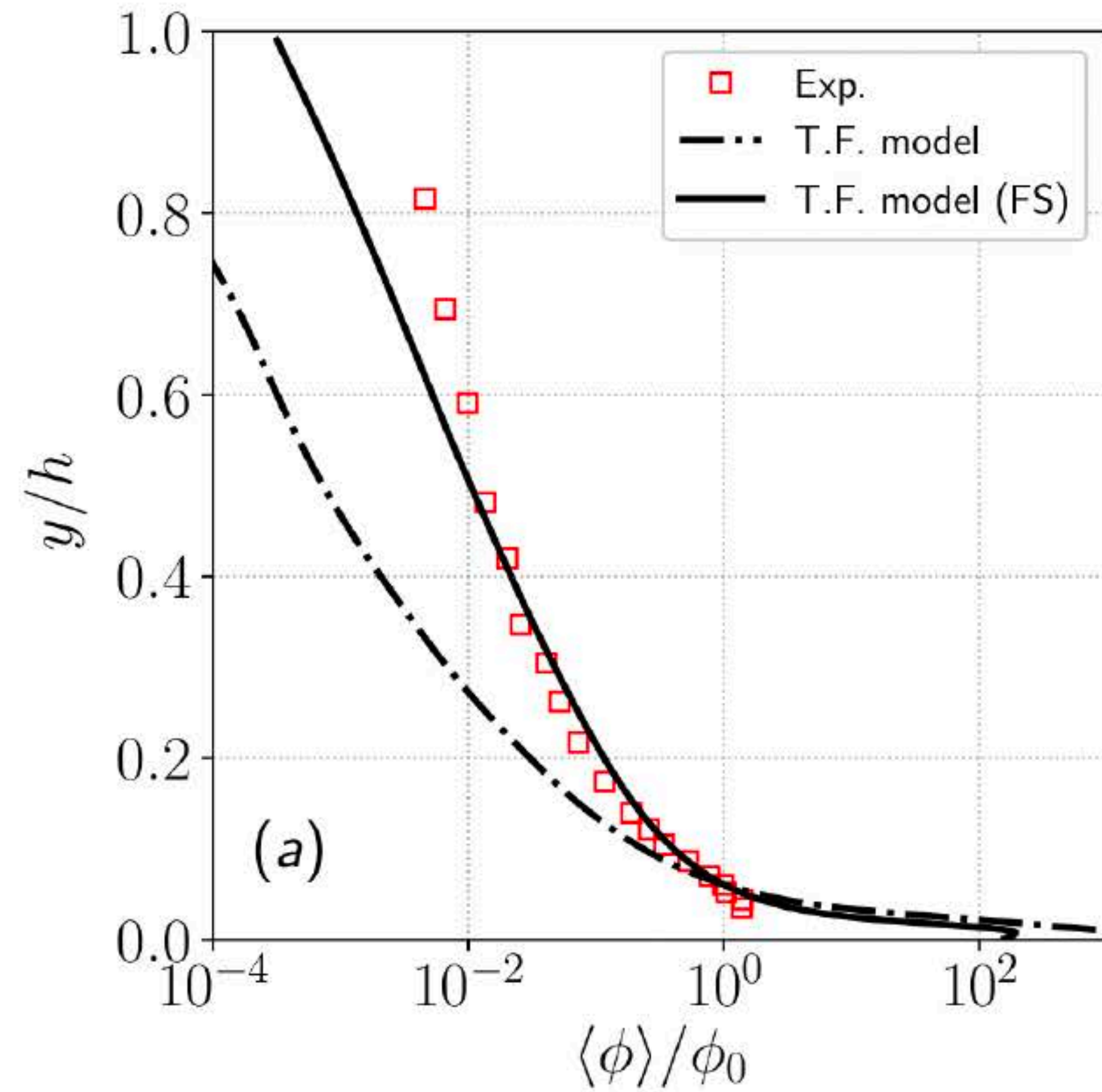
Filtered particle relaxation time

$$\tilde{t}_s = f \left(\|\tilde{u}_i^f - \tilde{u}_i^s\|, \nu^f + \nu_p^t \right)$$

Eddies smaller than d_p modify the particle response time by increasing the viscosity “seen” by the particles

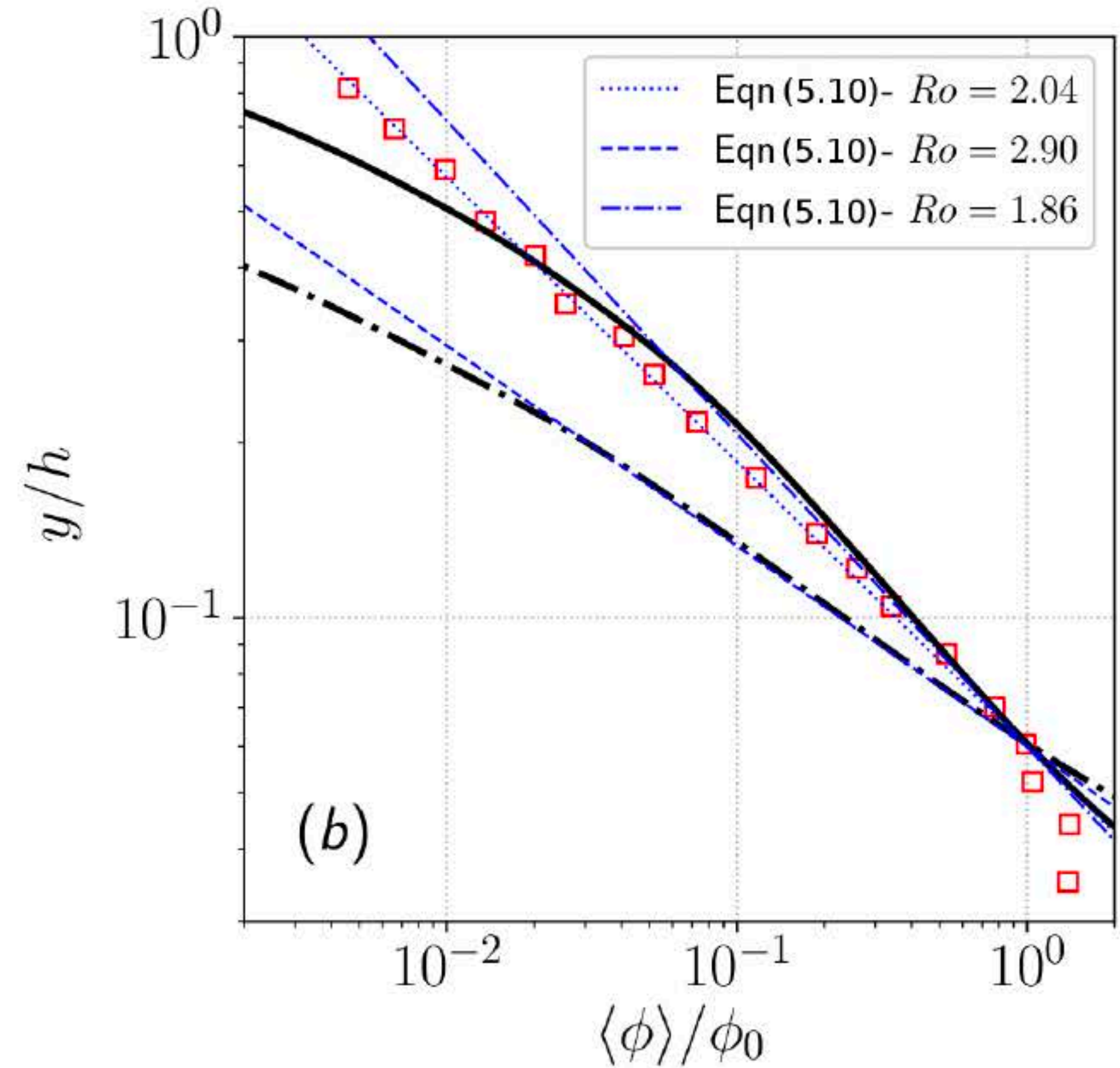
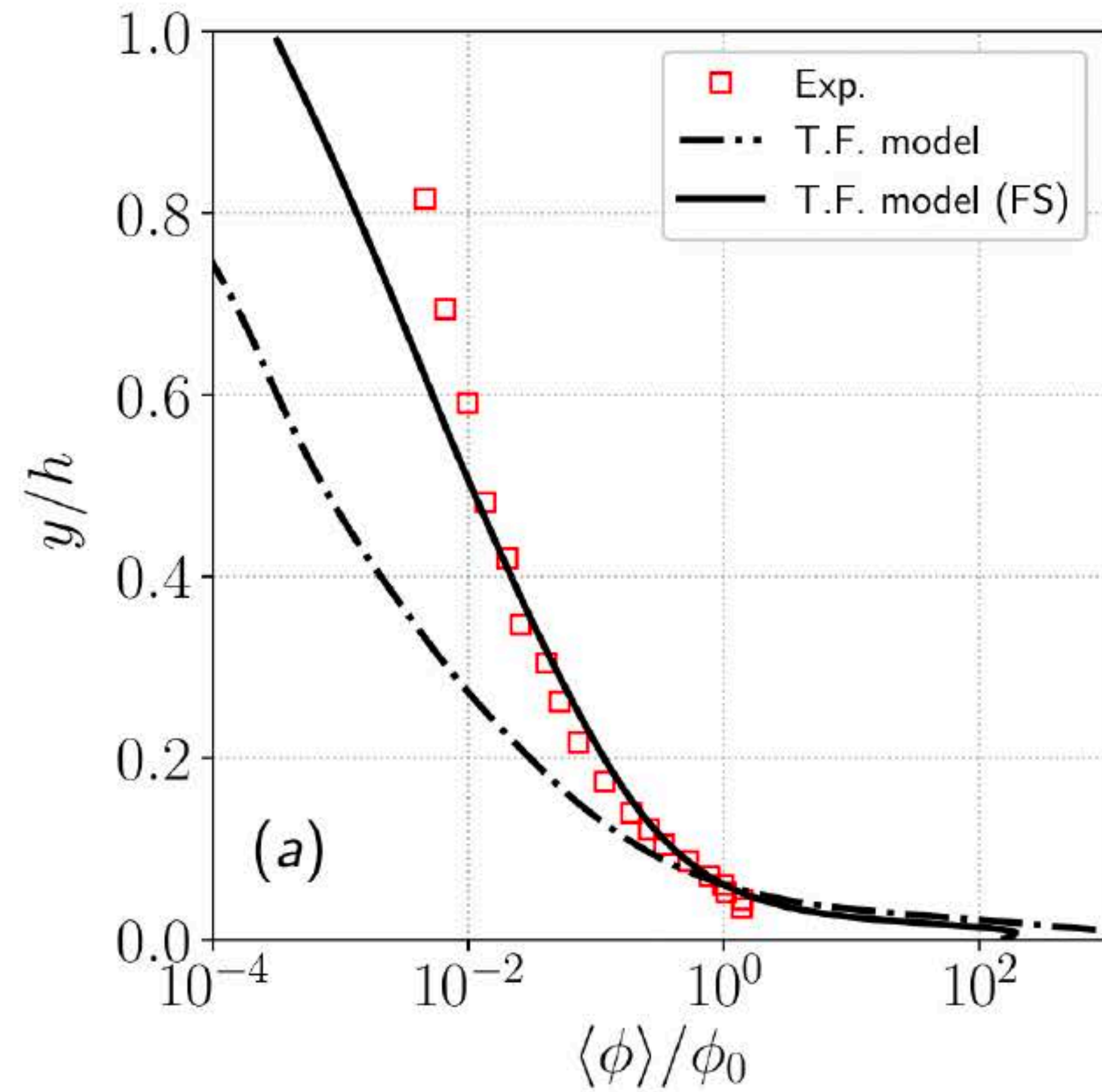
- Effective turbulent viscosity at the particle scale: $\nu_p^t \approx u_f' d_p \approx \varepsilon_p^{1/3} d_p^{4/3}$

Finite-size correction model for two-fluid LES



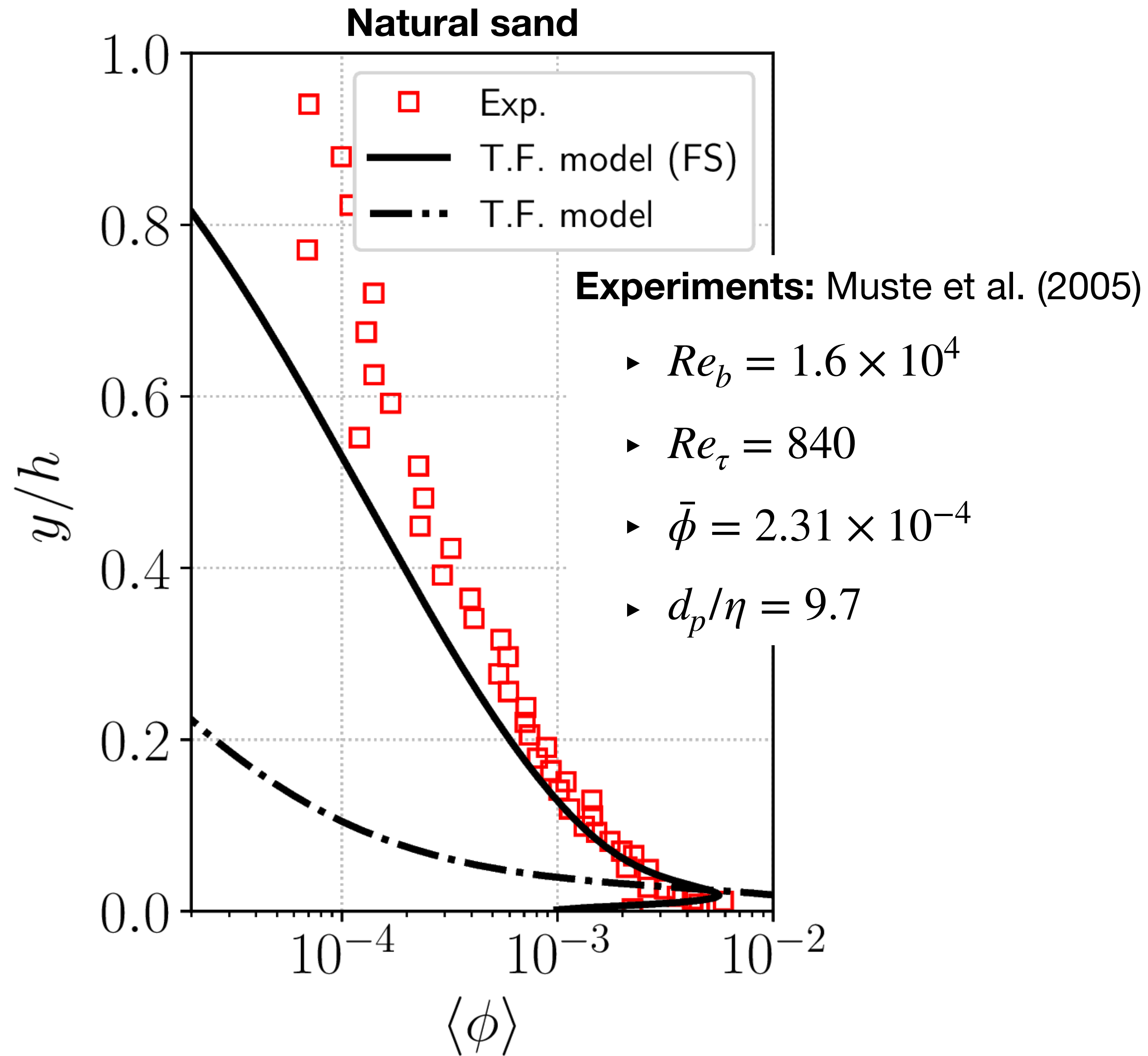
Eqn (5.10): $\frac{c}{c_r} = \left(\frac{z_r}{z} \right)^{R_o}$ with Prandtl mixing length: $\epsilon_s = \kappa u_* z$ and $R_o = \frac{w_s}{\kappa u_*}$ the Rouse number

Finite-size correction model for two-fluid LES

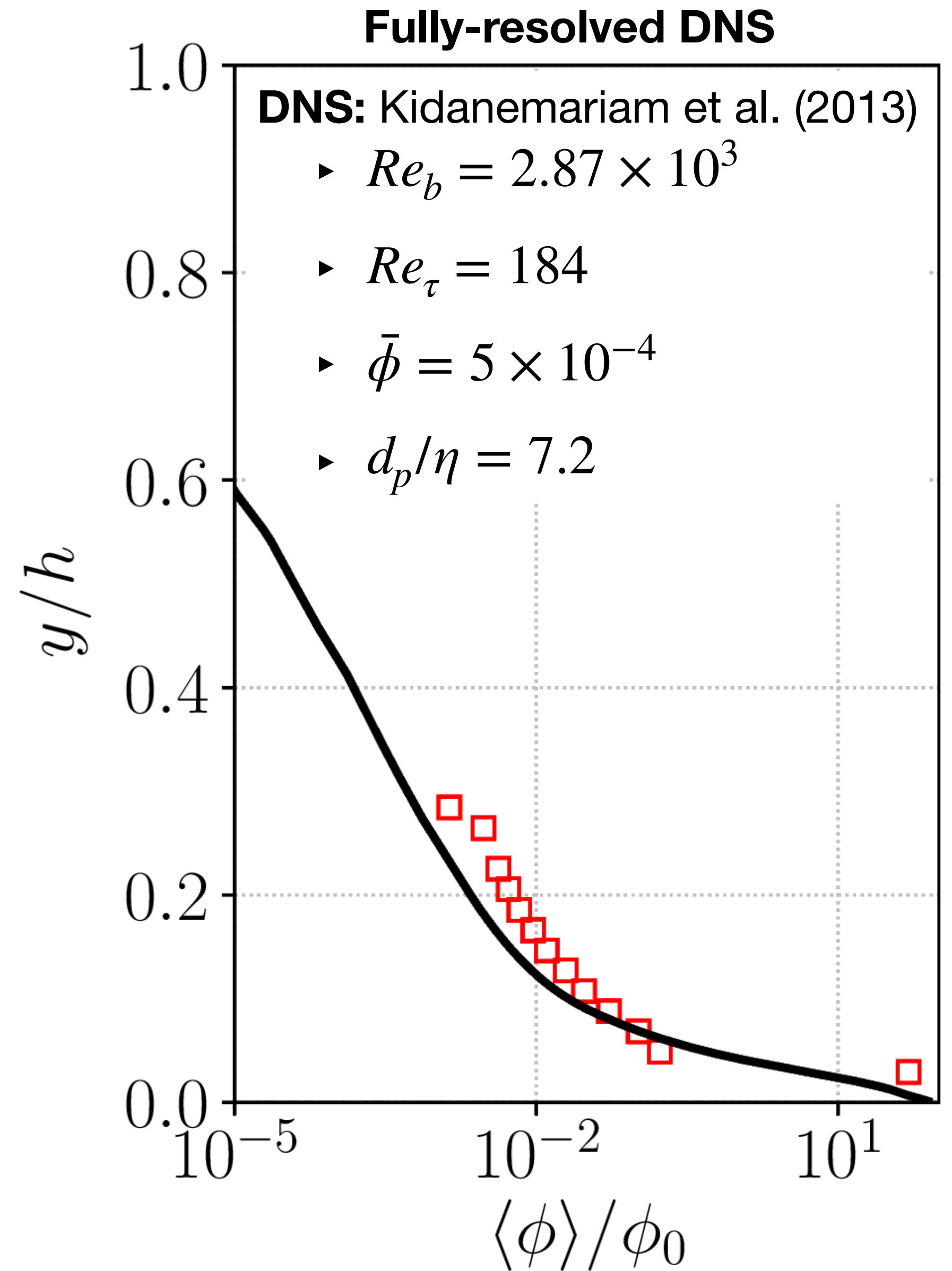
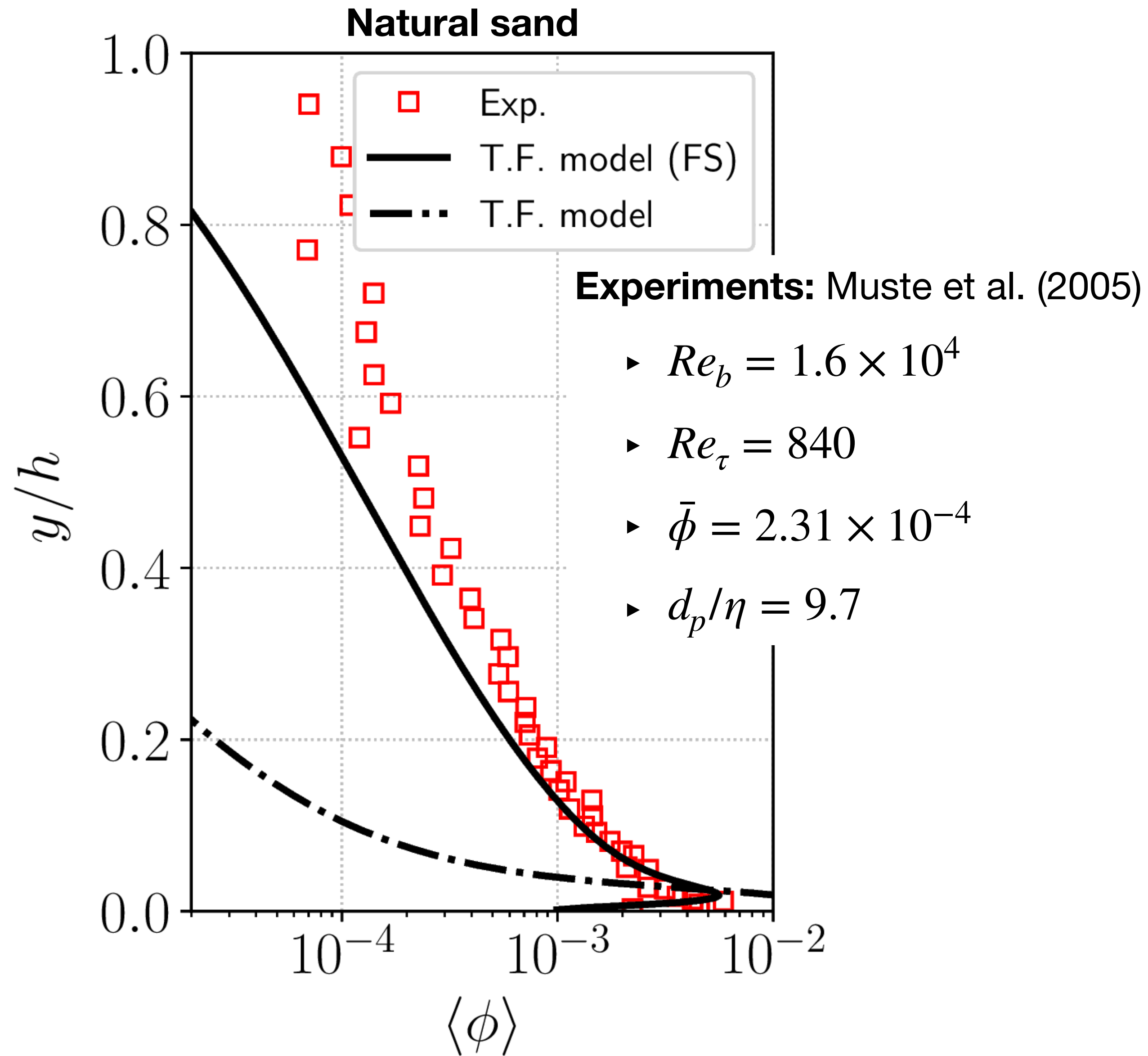


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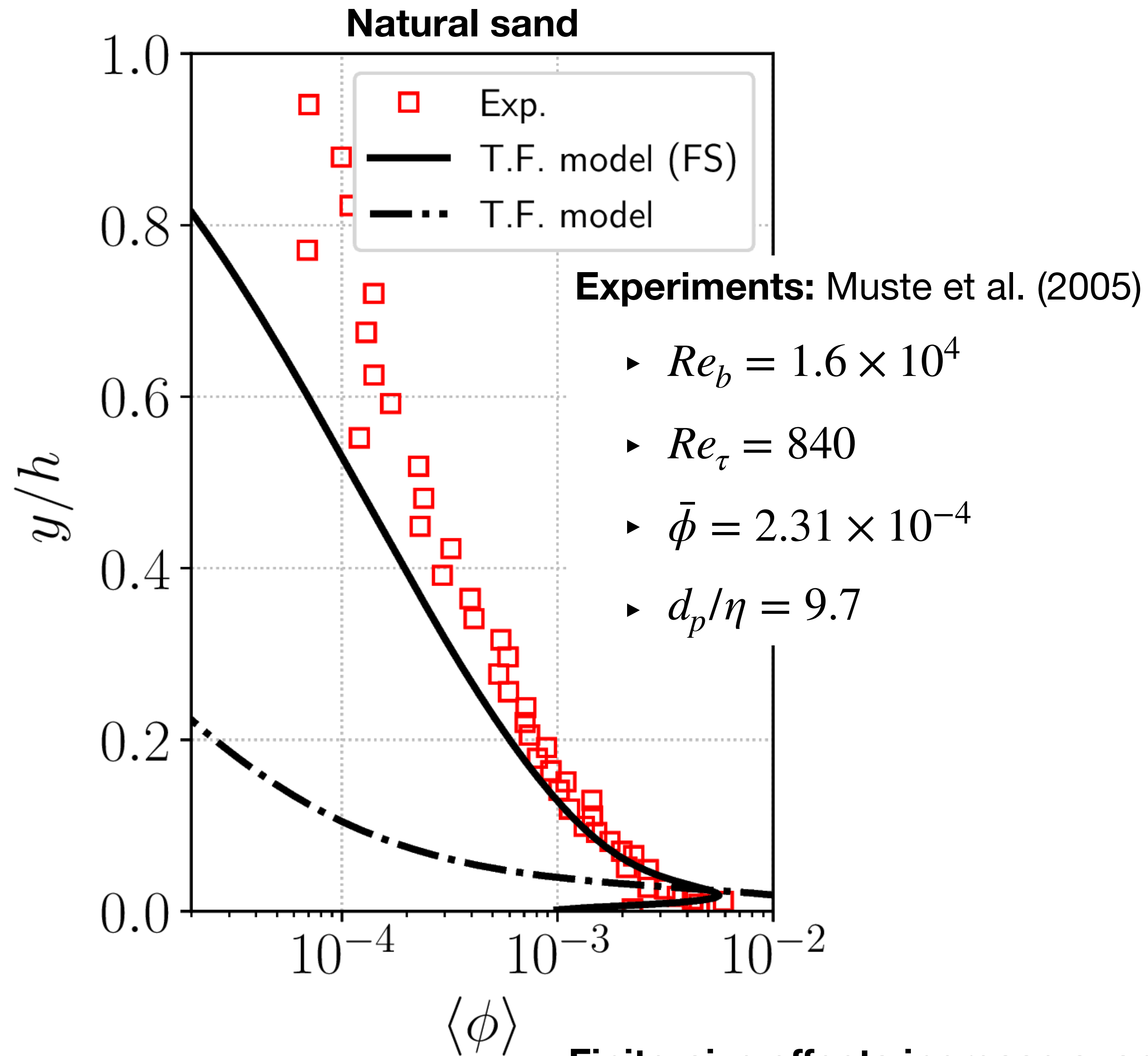
Finite-size correction model for two-fluid LES



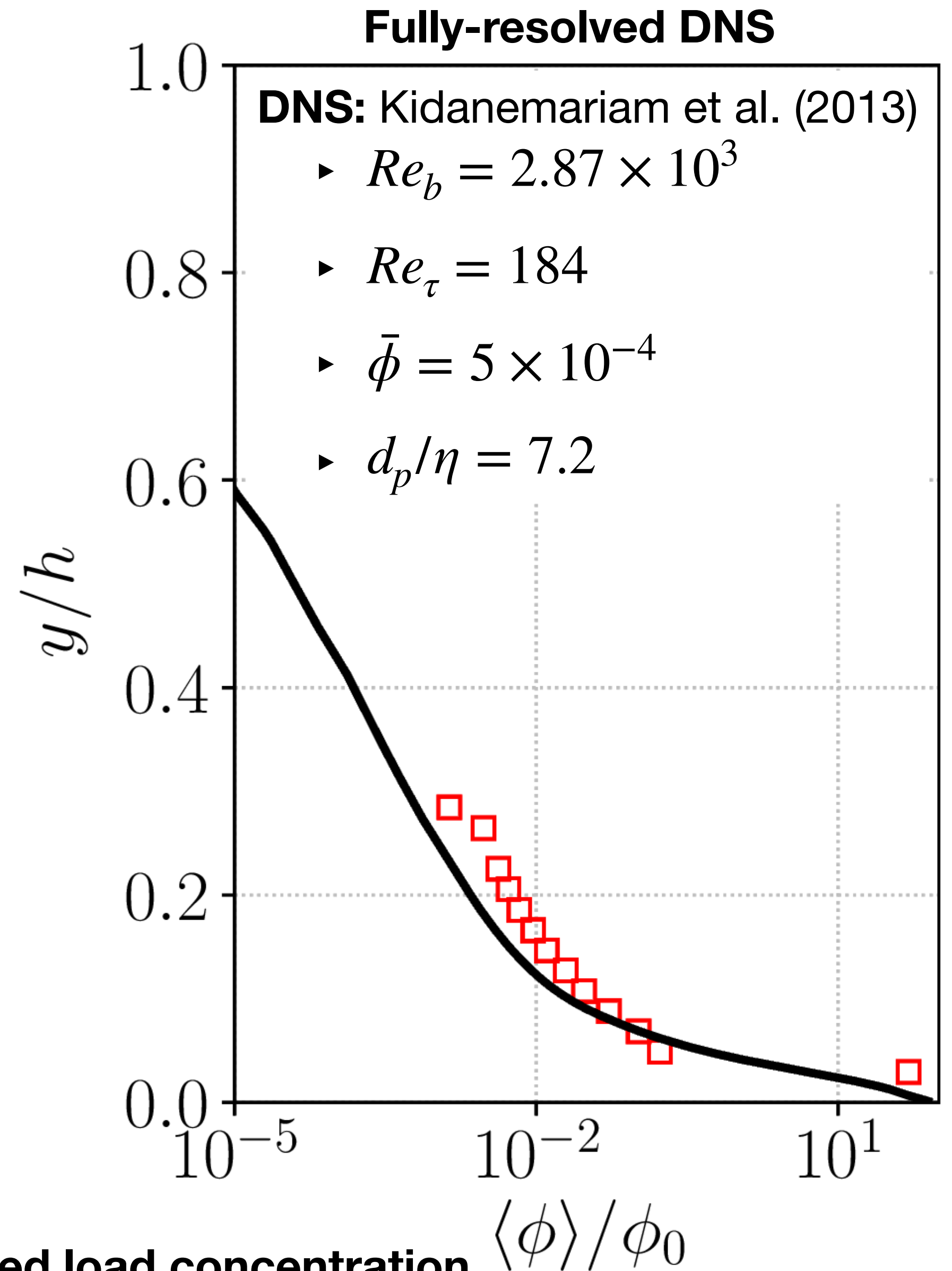
Finite-size correction model for two-fluid LES



Finite-size correction model for two-fluid LES



Finite-size effects increase suspended load concentration



WHAT COULD EXPLAIN $S_c < 1$?

Sediment mass balance:

$$W_s \langle \phi \rangle + \frac{\nu^t}{S_c} \frac{d\langle \phi \rangle}{dy} = 0$$

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$$S_c < 1 \iff W_s \downarrow$$

The turbulent Schmidt number is not smaller than unity
BUT
settling velocity is reduced due to finite-size effects

Other evidence of settling retardation are presented in Chauchat et al. (PRF 2022)

Conclusions

Finite size effects on suspended-load

- ➡ They are significant for medium sand particles
- ➡ They can be modeled using a sub grid scale viscosity in the drag force
- ➡ They might explain why the turbulent Schmidt number has been taken as $Sc < 1$

Perspectives

- ➡ Extend the approach to larger size ratios: $d_p/\eta > 10$
- ➡ Apply the two-fluid LES to wave-driven sand transport and ripples formation

sedFOAM team

LEGI
Univ. Grenoble Alpes
France



C. Bonamy



E. Puig-Montella



R. Chassagne

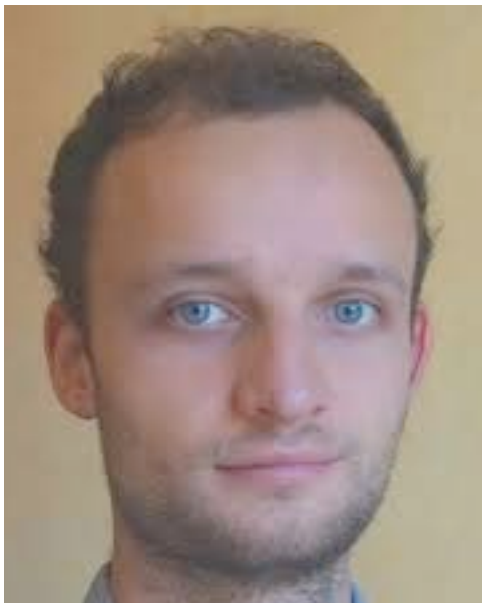


A. Gilletta



H. Divel

CACR
Univ. Delaware
USA



A. Mathieu



T. Nagel



T. Revil-Baudard

...



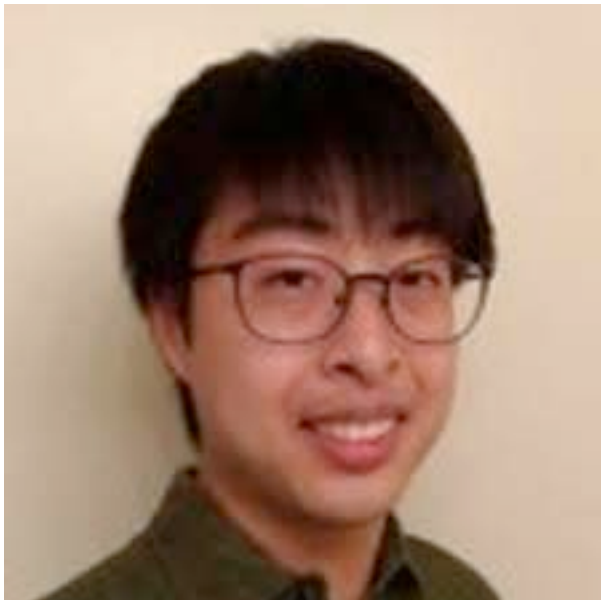
T.-J. Hsu.



Z. Cheng



A. Salimi Tarazouj



B. Tsai

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