Coupling RANS and LES approaches to limit computational cost

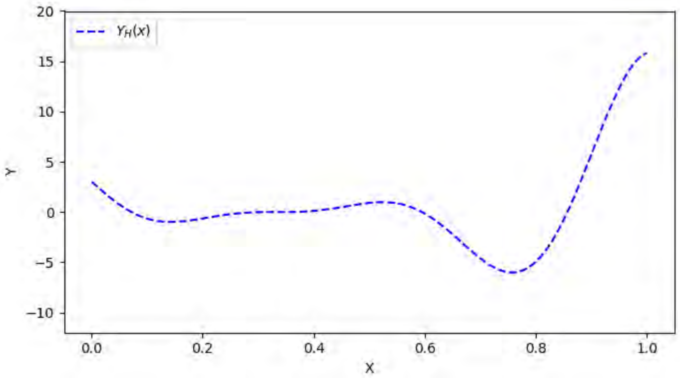
T. Berthelon - G. Balarac - A. Mahdi

LEGI/MOST

05/12/2024

Part I :
Multi-fidelity for turbulent flow

Multi-fidelity method



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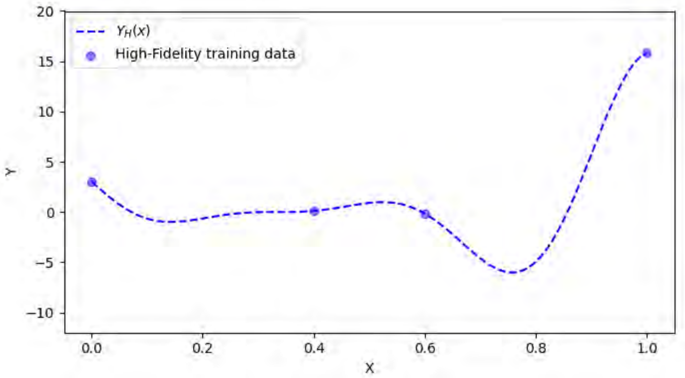

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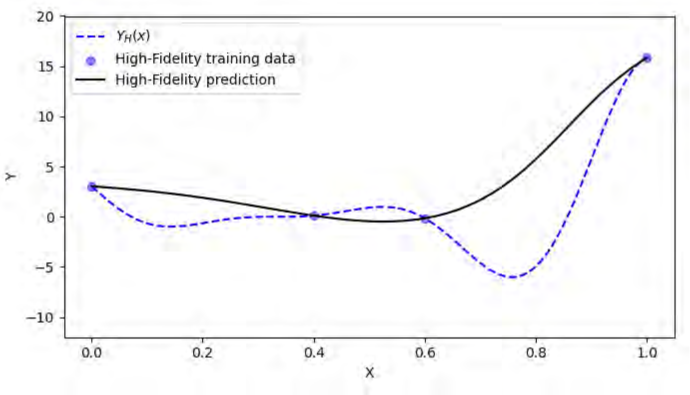

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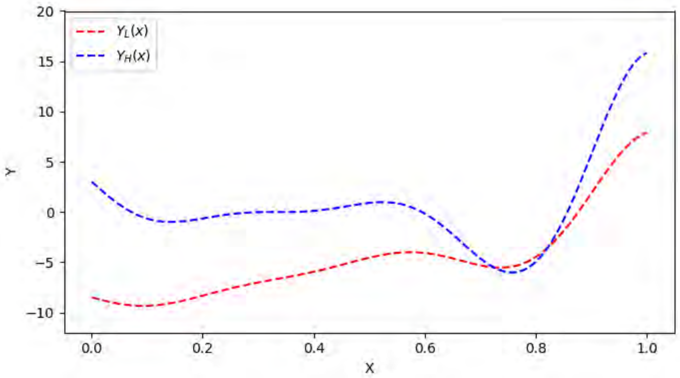

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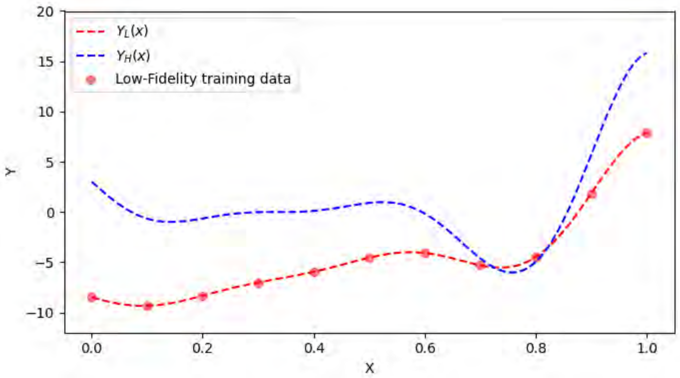

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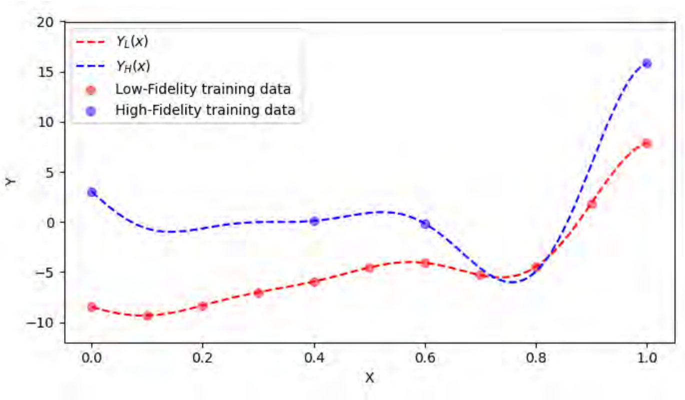

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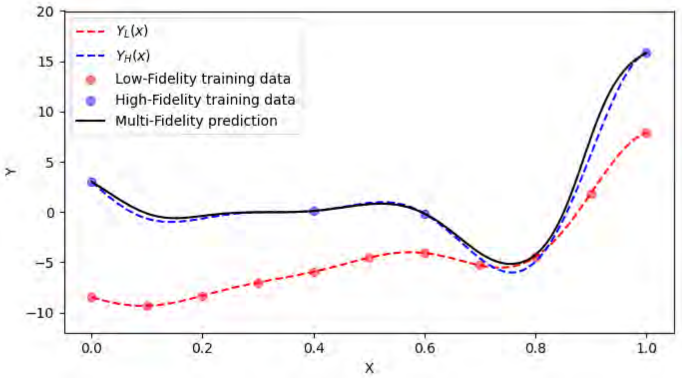

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Multi-fidelity with Artificial Neural Networks

The objective of the MF methods is to learn the relation: $\boxed{y_H = f(x, y_L)}$

A. Mole et al, "Multi-fidelity surrogate modelling of wall mounted cubes," Flow, Turbulence and Combustion (2023), 10.1007/s10494-022-00391-1.

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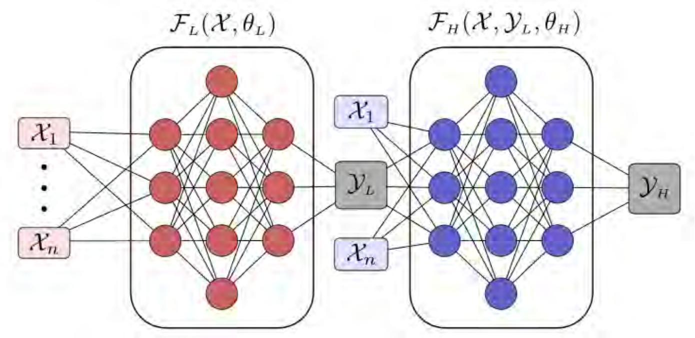

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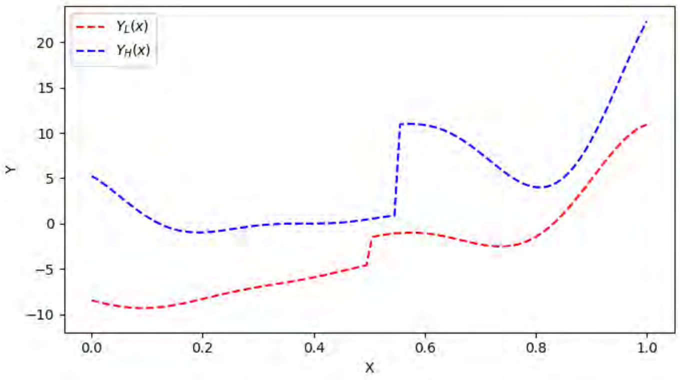
Discontinous case with shift

To take into account a potential shift between y_H and y_F, we add an hyperparameter τ which is selected with LOOCV : $\boxed{y_H = f(x, y_L(x-\tau))}$



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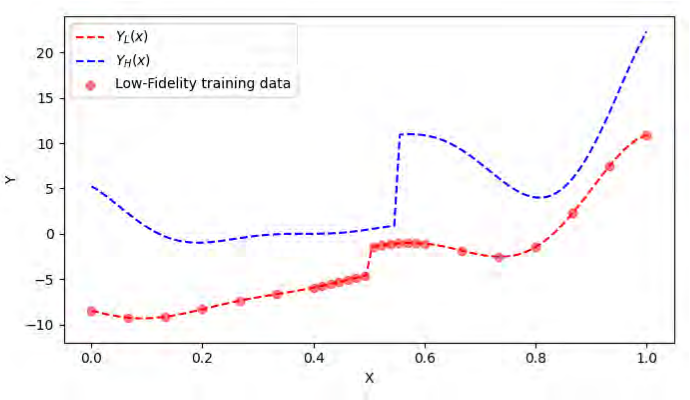

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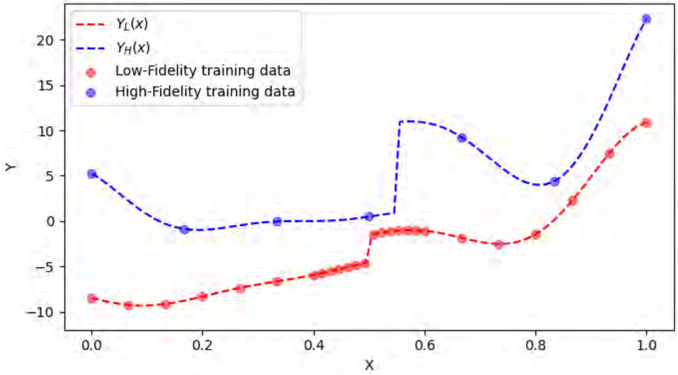

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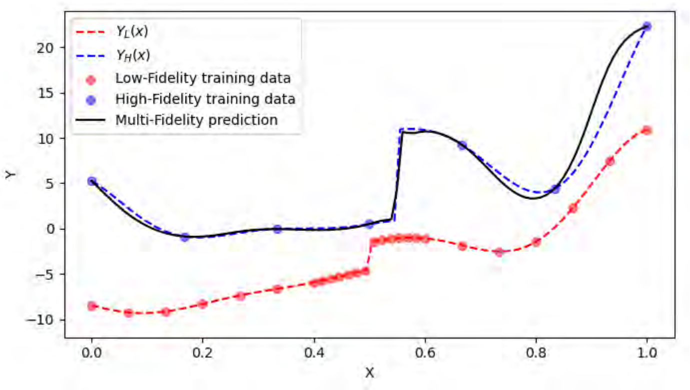
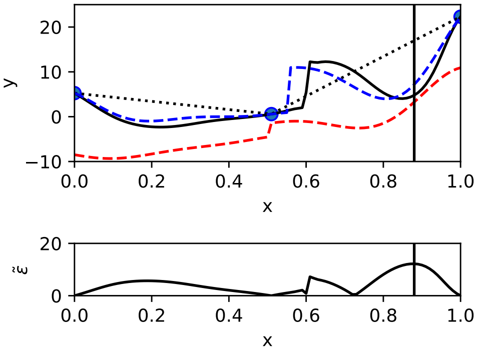
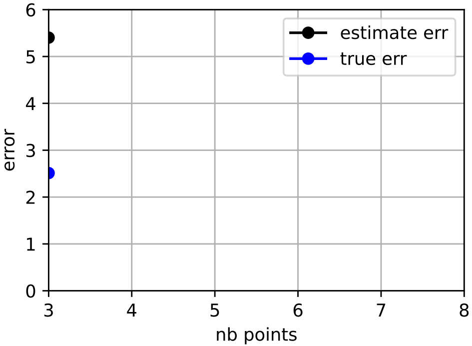


Strategy to select high-fidelity points

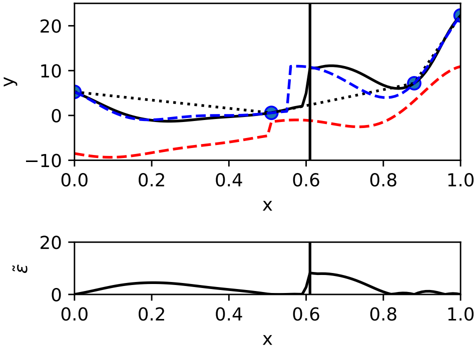
3 first points : $x_{min}, x_{max}, \arg\max_x \left(\left| \frac{\partial^2 y_L(x)}{\partial^2 x} \right| \right)$

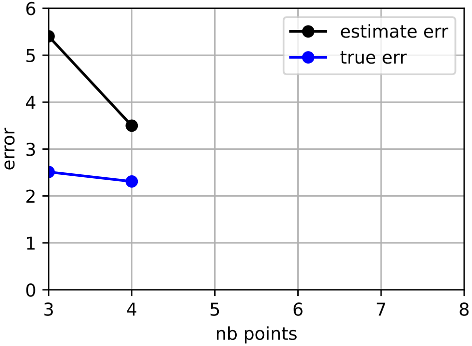




Strategy to select high-fidelity points

Next point : $\arg\max_x(\tilde{\epsilon})$





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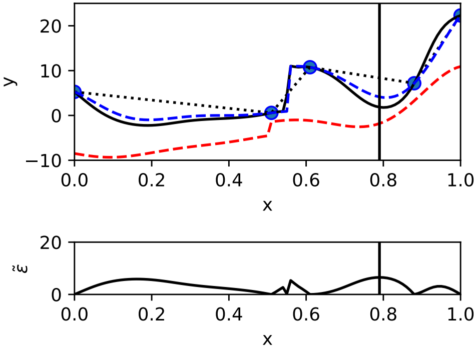

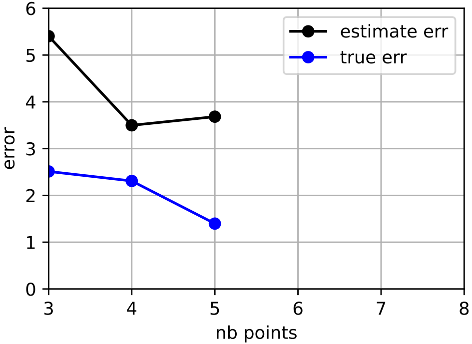

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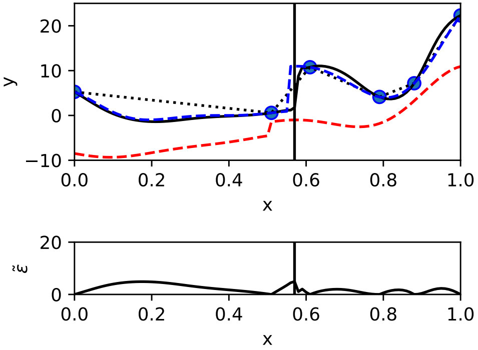

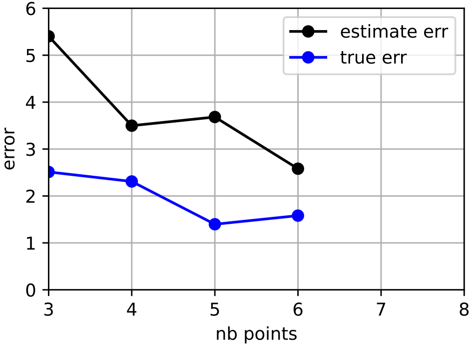

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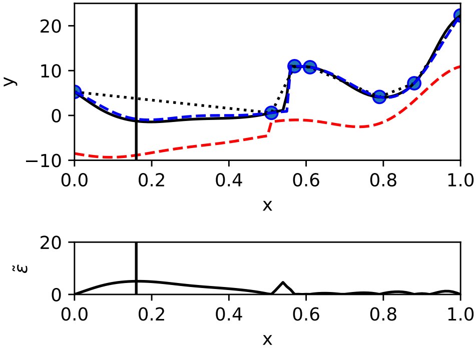

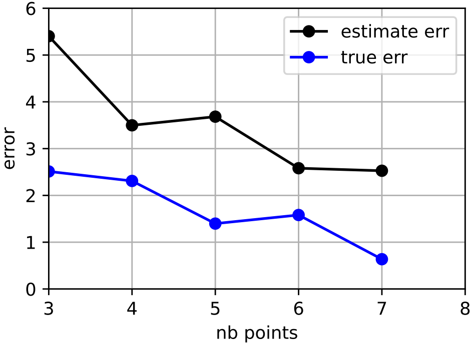

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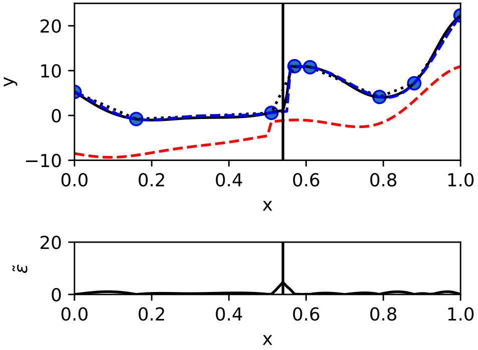
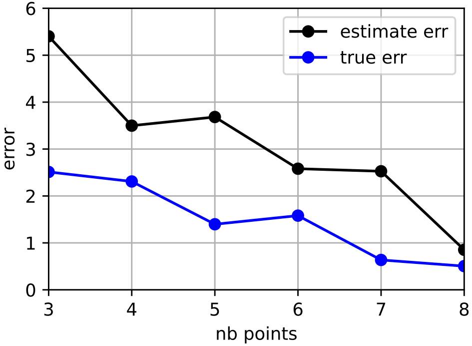
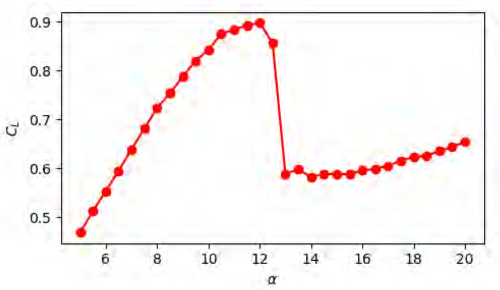




Lift coefficient of a NACA0012 ($Re_c = 50000$)

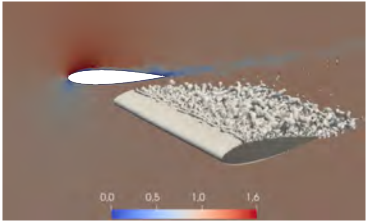
Numerical setup :

- YALES2 finite volume librairy
- Incompressible fractionnal-step solver
- Unstructred Mesh
- Automatic mesh adapation
- RANS and LES

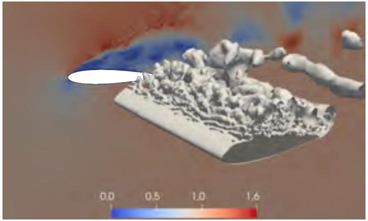


2D-RANS simulation

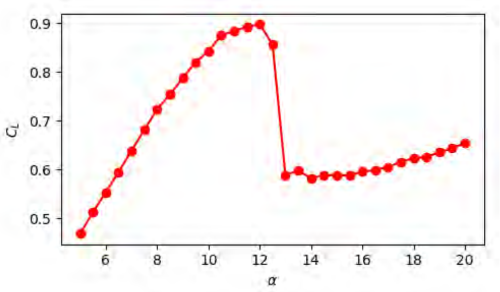
Lift coefficient of a NACA0012 ($Re_c = 50000$)



LES, $\alpha = 8^\circ$

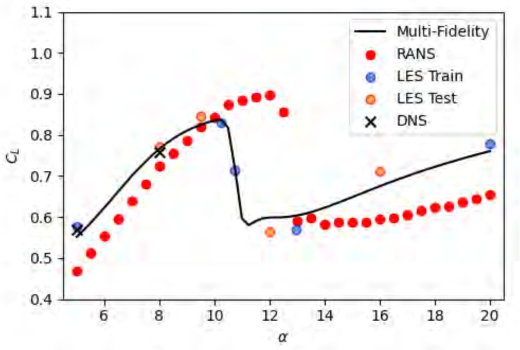


LES, $\alpha = 16^\circ$



2D-RANS simulation

Strategy applied on RANS and LES NACA0012 case ($Re_c = 50000$)



$\Rightarrow$ Only 5 LES evaluations are needed to capture the main characteristics of the target function.

Conclusion and Perspective

Conclusions :

- Strategy seems relevant
- RANS can help to perform LES
- ML used to help user

Difficulty :

- Sensitivity to network parameters

Conclusion and Perspective

Conclusions :

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- RANS can help to perform LES
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Difficulty :

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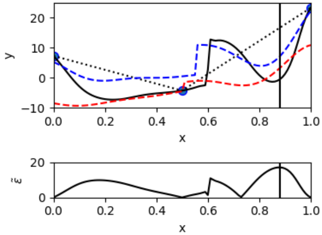
Perspective :

- Taking into account the uncertainty of statistical convergence



Part II :

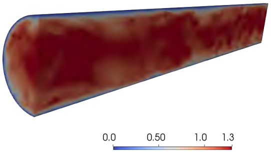
Hybrid RANS-LES method based on rigourous velocity separation

Decomposition of the velocity

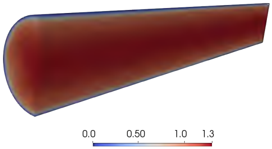
LES velocity decomposition : $u_i = \overline{u_i} + \tilde{u}_i$

Using temporal mean : $\boxed{u_i = \langle u_i \rangle + \overline{u_i}' + \tilde{u}_i}$

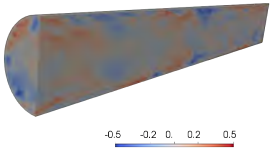
With $\langle \tilde{u}_i \rangle = 0$, and therefore $\langle u_i \rangle = \langle \overline{u_i} \rangle$



$\overline{u_1}$



$\langle u_1 \rangle$



$\overline{u_1}'$

Coupled equations

Playing with equation leads to 2 coupled equations :

- Equation on the mean field (RANS):

$$\frac{\partial}{\partial x_j}\left(\langle u_i\rangle\langle u_j\rangle\right) = -\frac{\partial\langle P\rangle}{\partial x_i} + \nu\frac{\partial^2\langle u_i\rangle}{\partial x_j\partial x_j} - \frac{\partial}{\partial x_j}\langle u_i' u_j'\rangle$$

- Equation on the fluctuation (LES) :

$$\frac{\partial\overline{u}_i'}{\partial t} + \frac{\partial}{\partial x_j}\left(\overline{u}_i'\overline{u}_j' + \langle u_i\rangle\overline{u}_j' + \overline{u}_i'\langle u_j\rangle\right) = -\frac{\partial\overline{P}'}{\partial x_i} + \nu\frac{\partial^2\overline{u}_i'}{\partial x_j\partial x_j} - \frac{\partial}{\partial x_j}\langle\tilde{u}_i\tilde{u}_j\rangle + \frac{\partial}{\partial x_j}\langle u_i' u_j'\rangle$$

Moerover,

$$\langle u_i' u_j'\rangle = \langle\overline{u}_i'\overline{u}_j'\rangle + \langle\tilde{u}_i\tilde{u}_j\rangle$$

Labourasse, E., & Sagaut, P. (2004). Advance in RANS-LES coupling, a review and an insight on the NLDE approach. Archives of Computational methods in Engineering, 11, 199-256.
Xiao, H., & Jenny, P. (2012). A consistent dual-mesh framework for hybrid LES/RANS modeling. Journal of Computational Physics, 231(4), 1848-1865.

Hybrid RANS-LES

The idea : Solving the fluctuation requires less element than solving the complete field

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Description of the method :

1. Compute steady field with RANS modeling
2. Compute fluctuation using steady field from RANS

Hybrid RANS-LES

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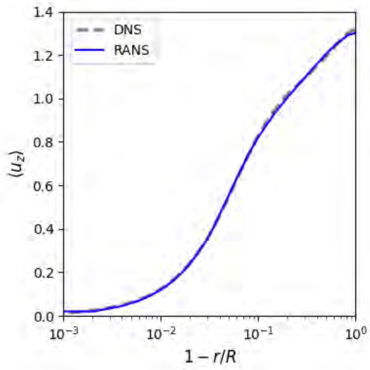
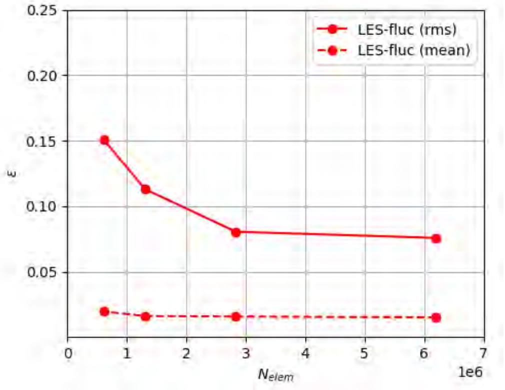


Axial velocity in turbulent pipe ($Re_D = 5300$)

Mesh convergence - Pipe flow ($Re_D = 5300$)

Automatic mesh convergence based on physical crietrion



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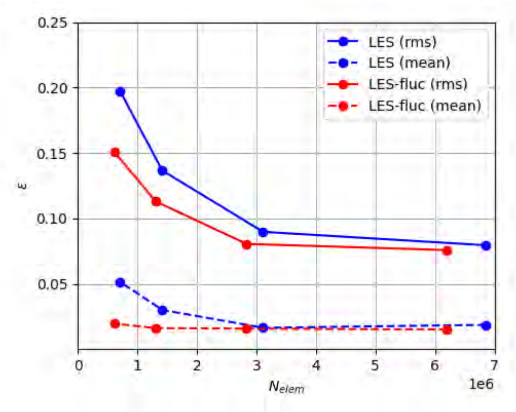

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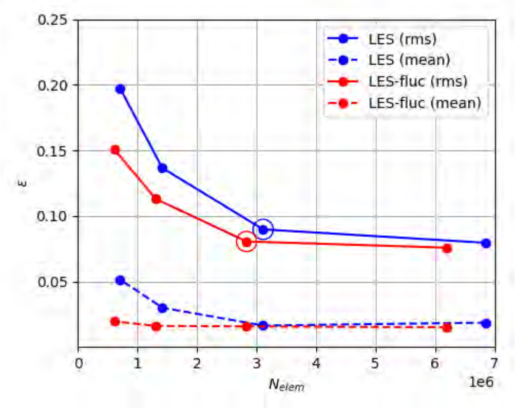
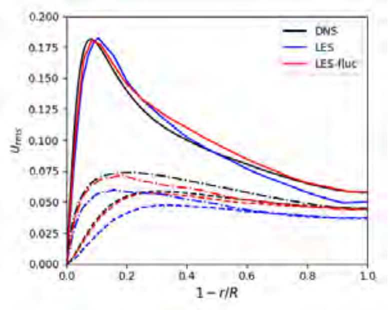
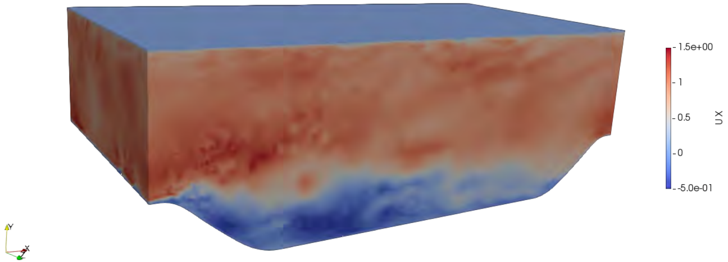




$\Rightarrow$ For a same level of error, the LES-fluc has less element than classical LES

Periodic Hill ($Re_D = 5600$)



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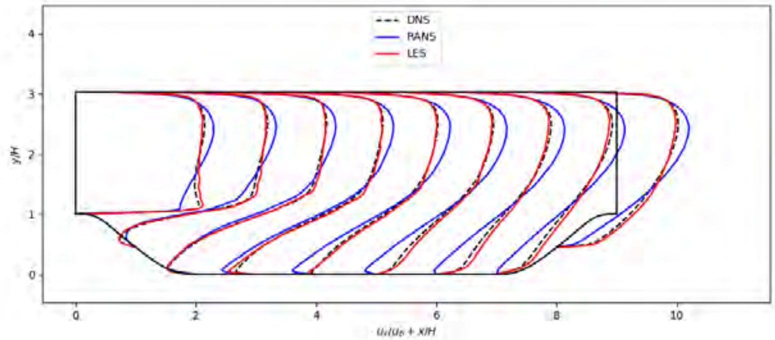

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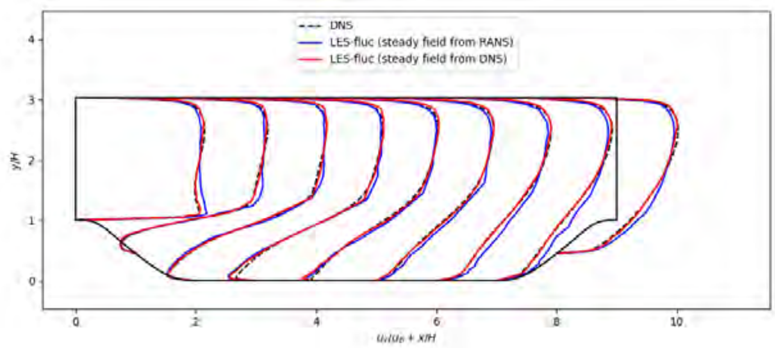


$\Rightarrow$ the time average of the fluctuation corrects the RANS field

Conclusion and Perspective

Conclusions :

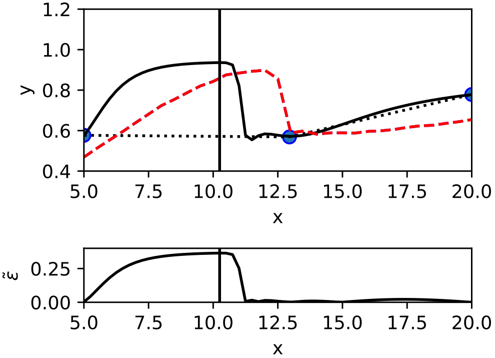
- Potential gain on number of elements
- The fluctuation corrects the RANS field

Perspectives :

- Consolidate the gain on number of elements
- Zonal approach ?
- Use Reynolds stress from fluctuation to correct RANS ?

Merci pour votre attention !

Strategy applied on RANS and LES NACA0012 case



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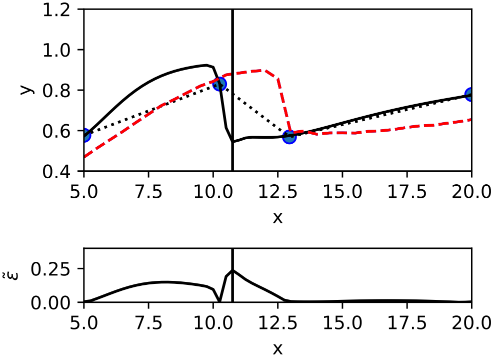

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